Mapping Parametrized Difference Revision Operators to Belief Contraction

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Structure of Presentation

- ✓ Belief Change
- ✓ The AGM Paradigm
- ✓ Revision and Contraction
- ✓ Interdefinability of Operations
- ✓ Parametrized Difference Revision Operators (PDR Operators)
- ✓ PDR Operators in the Realm of Contraction
- ✓ Conclusions

BELIEF CHANGE (REVISION) [1/2]

- Mary has just discovered that George and Dimitra *are not* her true parents.
- She was adopted when she was 6 months old from an orphanage in Sao Paolo.
- The news really shook Mary.
- Much of what she used to believe about herself and her family was wrong.
- She must, now, put her thoughts back in order.

Belief Change (Revision) [2/2]

- A typical (although rather dramatic) instance of a belief-revision scenario.

- A *rational* agent receives new (contradicting) information, that makes her change her beliefs.

- *Withdraw* some of the old beliefs, before she can (consistently) accommodate the new information.

- Accept the *consequences* that might result from the interaction of the new information with the old beliefs.

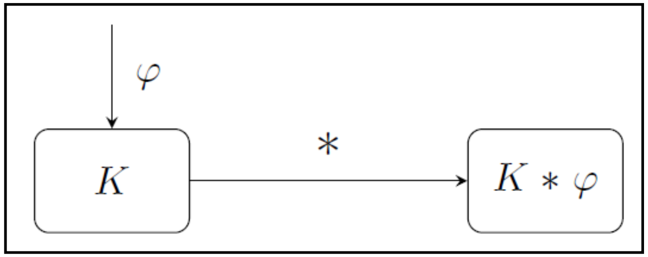
THE AGM PARADIGM [1/2]

- The study of Belief Change can be traced back to the early ’80s, with the seminal work of Alchourrón, Gärdenfors, and Makinson.
- They established the *AGM paradigm*; to this date, the dominant framework in Belief Change.
- Beliefs are modeled as *sentences* (φ, ψ) of a propositional language.
- Belief sets (K) are modeled as *sets of sentences closed* under logical implication (theories).
- The process of belief change is encoded as a *function* that maps a theory and a sentence to a new theory.

The AGM Paradigm [2/2]

- The AGM paradigm studies both *belief revision* and *belief contraction*.
- Belief Revision: Incorporation of a sentence that is inconsistent with a belief set.
- Belief Contraction: Withdrawal of a sentence from a belief set.
- The AGM paradigm characterizes *rational* belief revision and belief contraction functions, by means of a set of rationality postulates for each case.
- Principle of Minimal Change: The new belief set differs *as little as possible* from the old belief set.

The AGM Postulates for Revision



(**K** $*$ **1**) $K * \varphi$ is a theory of $\mathcal{L}$.

(**K** $*$ **2**) $\varphi \in K * \varphi$.

(**K** $*$ **3**) $K * \varphi \subseteq K + \varphi$.

(**K** $*$ **4**) If $\neg\varphi \notin K$, then $K + \varphi \subseteq K * \varphi$.

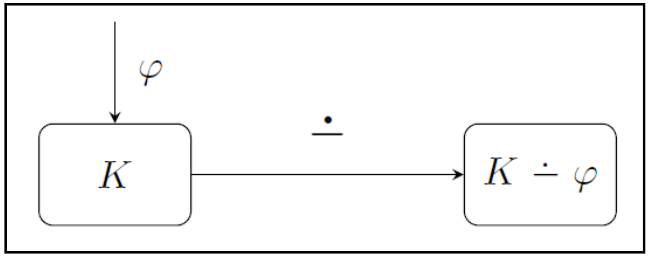
(**K** $*$ **5**) If φ is consistent, then $K * \varphi$ is also consistent.

(**K** $*$ **6**) If $\varphi \equiv \psi$, then $K * \varphi = K * \psi$.

(**K** $*$ **7**) $K * (\varphi \wedge \psi) \subseteq (K * \varphi) + \psi$.

(**K** $*$ **8**) If $\neg\psi \notin K * \varphi$, then $(K * \varphi) + \psi \subseteq K * (\varphi \wedge \psi)$.

THE AGM POSTULATES FOR CONTRACTION



(K $\dot{-}$ 1) $K \dot{-} \varphi$ is a theory of $\mathcal{L}$.

(K $\dot{-}$ 2) $K \dot{-} \varphi \subseteq K$.

(K $\dot{-}$ 3) If $\varphi \notin K$, then $K \dot{-} \varphi = K$.

(K $\dot{-}$ 4) If $\nvDash \varphi$, then $\varphi \notin K \dot{-} \varphi$.

(K $\dot{-}$ 5) If $\varphi \in K$, then $K \subseteq (K \dot{-} \varphi) + \varphi$.

(K $\dot{-}$ 6) If $\varphi \equiv \psi$, then $K \dot{-} \varphi = K \dot{-} \psi$.

(K $\dot{-}$ 7) $(K \dot{-} \varphi) \cap (K \dot{-} \psi) \subseteq K \dot{-} (\varphi \wedge \psi)$.

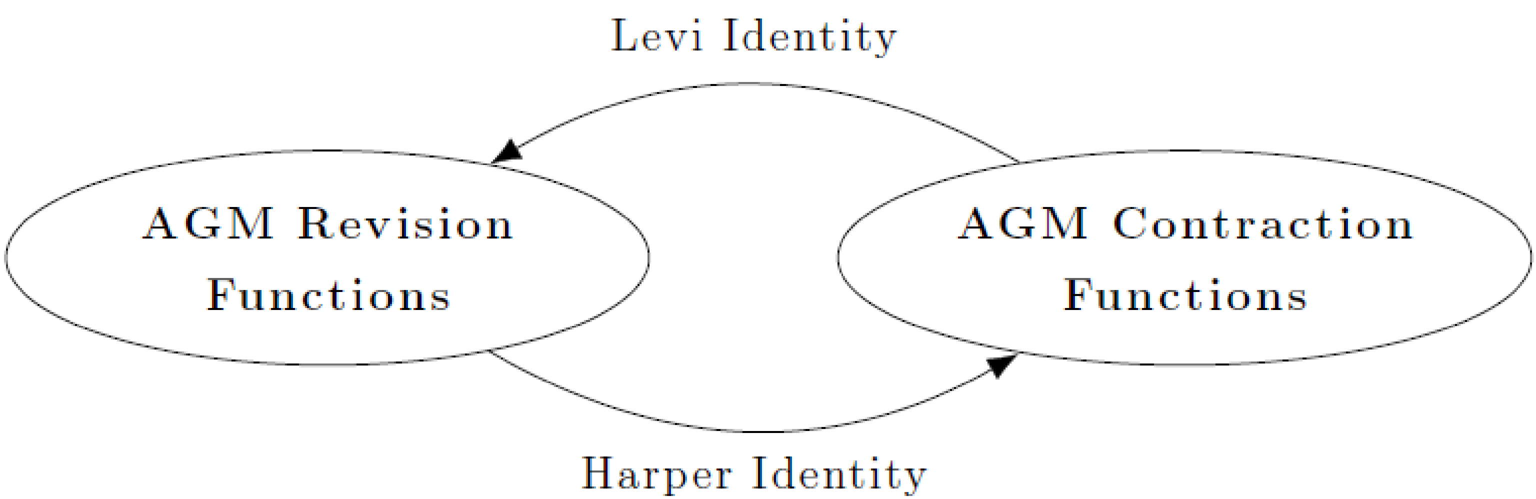
(K $\dot{-}$ 8) If $\varphi \notin K \dot{-} (\varphi \wedge \psi)$, then $K \dot{-} (\varphi \wedge \psi) \subseteq K \dot{-} \varphi$.

INTERDEFINABILITY OF OPERATORS

- Revision and contraction can be defined in terms of each other through **Levi** and **Harper Identities**.

Levi Identity: $K * \varphi = (K \dot{-} \neg\varphi) + \varphi$

Harper Identity: $K \dot{-} \varphi = (K * \neg\varphi) \cap K$



Concrete Revision Operators

- Several concrete “off the self” revision functions (operators), implementing the process of belief revision, have been proposed.

- Among the well-known proposals, only Dalal’s revision operator satisfies the full set of AGM postulates for revision (K*1) – (K*8).

- Simple and intuitive construction.

Parametrized Difference Revision Operators

- Introduced by Peppas and Williams (2016).
- Satisfy the AGM postulates for revision.
- Natural generalizations of Dalal's revision operator, with a much *greater range of applicability*.
- Low *representational* and *computational* cost, that makes them ideal for real-world applications.
- Have been axiomatically defined, very recently, in the realm of revision.
- We provide an axiomatic characterization *in the realm of contraction*.

PDR Operators in the Realm of Revision [1/2]

- PDR operators have been axiomatically defined in the realm of revision by Peppas and Williams (2018).

(D1) If $A \leqq_K B$, then $|A| \leq |B|$.

(D2) If $A \leqq_K B$, $p \leqq_K q$ and $q \notin B$, then $Ap \leqq_K Bq$.

(D3) If $A \leqq_K B$, $p \prec_K q$ and $q \notin B$, then $Ap \prec_K Bq$.

(D4) If $A \prec_K B$, $p \in K$, $q \notin B$ and for all $z \in B$, $z \leqq_K q$, then $Ap \prec_K Bq$.

(D5) If $p \leqq_K q$, $x \in \{p, \overline{p}\}$, $y \in \{q, \overline{q}\}$ and $x, y \in H$, then $x \leqq_H y$.

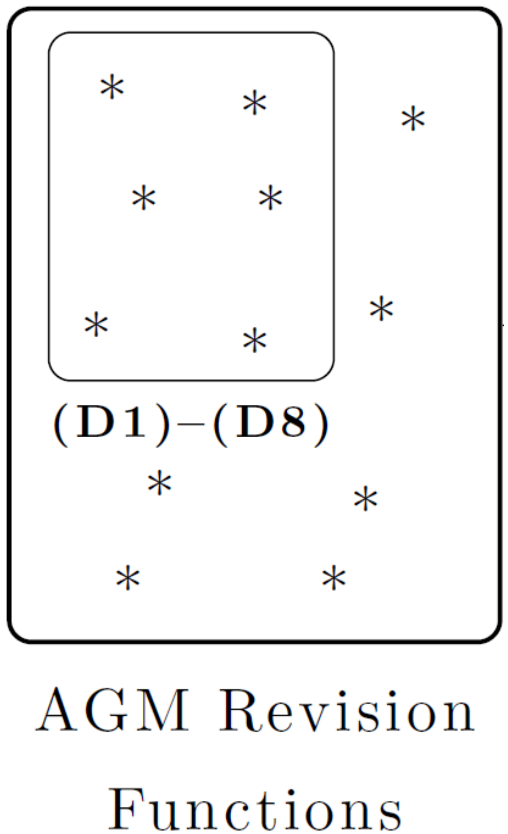
(D6) $K * \varphi = \bigcap_{w \in [\varphi * K]} w * \varphi$.

(D7) If $A \leqq_K E$, $B \subseteq K$, $\neg(\overline{A}B) \notin K * (\overline{A}B \vee \overline{C}D)$, C, $D \subseteq H$, and $\mathcal{L}_E = \mathcal{L}_C$, then $\neg(\overline{A}B) \notin (K \cap H) * (\overline{A}B) \vee (\overline{C}D)$.

(D8) If $A \prec_K E$, $B \subseteq K$, $\neg(\overline{C}D) \in K * (\overline{A}B \vee \overline{C}D)$, C, $D \subseteq H$, and $\mathcal{L}_E = \mathcal{L}_C$, then $\neg(\overline{C}D) \in (K \cap H) * (\overline{A}B) \vee (\overline{C}D)$.

PDR Operators in the Realm of Revision [2/2]

- PDR operators are a *proper subclass* of the whole class of AGM revision functions.



PDR Operators in the Realm of Contraction

- We provided the axiomatic characterization of PDR operators in the realm of *belief contraction*.
- Levi and Harper Identities were utilized.

(C1) If $A \leqq'_K B$, then $|A| \leq |B|$.

(C2) If $A \leqq'_K B$, $p \leqq'_K q$ and $q \notin B$, then $Ap \leqq'_K Bq$.

(C3) If $A \leqq'_K B$, $p \prec'_K q$ and $q \notin B$, then $Ap \prec'_K Bq$.

(C4) If $A \prec'_K B$, $p \in K$, $q \notin B$ and for all $z \in B$, $z \leqq'_K q$, then $Ap \prec'_K Bq$.

(C5) If $p \leqq'_K q$, $x \in \{p, \overline{p}\}$, $y \in \{q, \overline{q}\}$ and $x, y \in H$, then $x \leqq'_H y$.

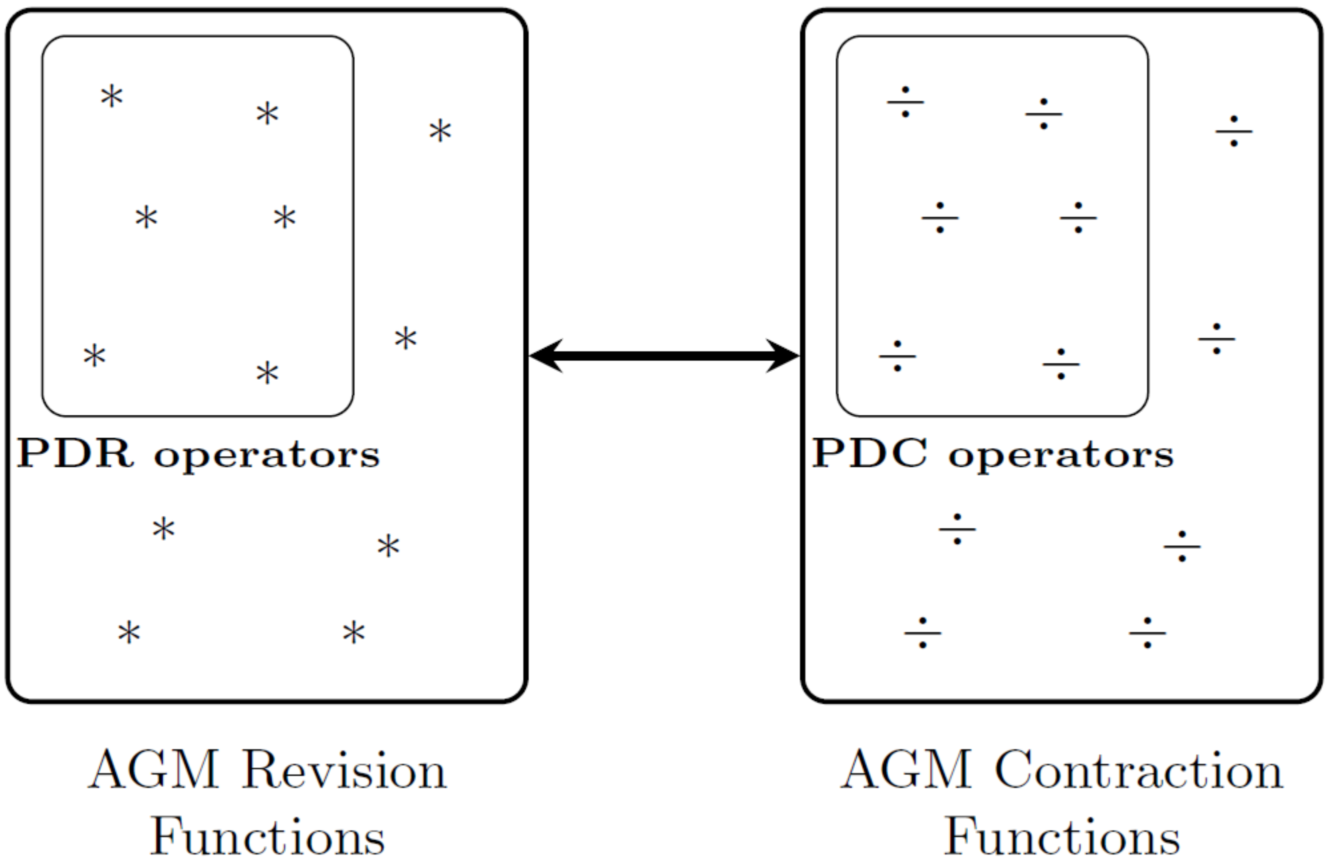
(C6) $K \div \varphi = K \cap \left(\bigcap_{w \in \left[(\neg\varphi \div \neg K) + K\right]} (w \div \varphi) + \neg\varphi \right)$.

(C7) If $A \leqq'_K E$, $B \subseteq K$, $\neg(\overline{A}B) \notin K \div \neg(\overline{A}B) \wedge \neg(\overline{C}D)$, $C, D \subseteq H$, and $\mathcal{L}_E = \mathcal{L}_C$, then $\neg(\overline{A}B) \notin (K \cap H) \div \neg(\overline{A}B) \wedge \neg(\overline{C}D)$.

(C8) If $A \prec'_K E$, $B \subseteq K$, $\neg(\overline{C}D) \in K \div \neg(\overline{A}B) \wedge \neg(\overline{C}D)$, $C, D \subseteq H$, and $\mathcal{L}_E = \mathcal{L}_C$, then $\neg(\overline{C}D) \in (K \cap H) \div \neg(\overline{A}B) \wedge \neg(\overline{C}D)$.

One-to-one Correspondence

Theorem 1. *Let $*$ and $\div$ be the AGM revision function and an AGM contraction function, respectively, that are connected via Levi and Harper Identities. Then, $\div$ satisfies (C1)–(C8) iff $*$ satisfies (D1)–(D8), respectively.*



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CONCLUSIONS

- PDR operators bring us a step closer to the development of a successful AGM belief-change system, due to their favorable properties.
- We have mapped PDR operators in the realm of belief contraction, by means of the Levi and Harper Identities, characterizing the class of PDC operators.
- The axiomatic characterization of this new class of operators has been completed for the two processes of belief change (revision and contraction).

Thank you!