On the Strong Version of Parikh's Relevance-Sensitive Axiom for Belief Revision

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Structure of Presentation

- ✓ Belief Revision
- ✓ The AGM Paradigm
- ✓ Parikh's Notion of Relevance
- ✓ Weak and Strong Version of Axiom (P)
- ✓ Properties of Faithful-Preorder Filterings
- ✓ Economy Due to Strong (P)
- ✓ Conclusions

Belief Revision [1/2]

- Mary has just discovered that George and Dimitra *are not* her true parents.
- She was adopted when she was 6 months old from an orphanage in Sao Paolo.
- The news really shook Mary.
- Much of what she used to believe about herself and her family was wrong.
- She must, now, put her thoughts back in order.

Belief Revision [2/2]

- A typical (although rather dramatic) instance of a belief-revision scenario.

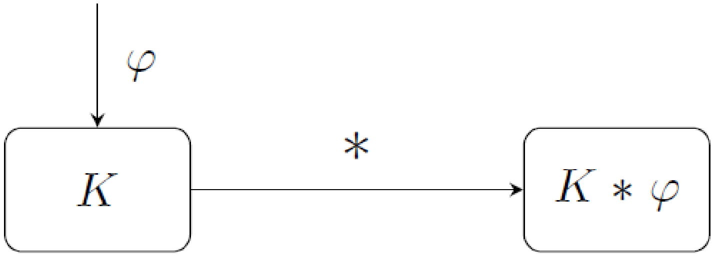
- A *rational* agent receives new (contradicting) information, that makes her change her beliefs.

- *Withdraw* some of the old beliefs, before she can (consistently) accommodate the new information.

- Accept the *consequences* that might result from the interaction of the new information with the old beliefs.

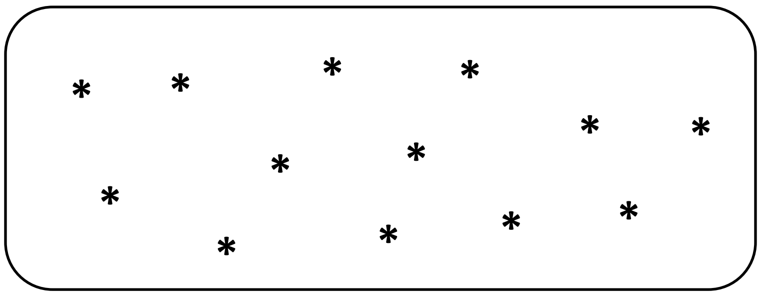
THE AGM PARADIGM

- The study of Belief Revision can be traced back to the early '80s, with the seminal work of Alchourrón, Gärdenfors, and Makinson.
- They established the *AGM paradigm*; to this date, the dominant framework in Belief Revision.
- Beliefs are modeled as *sentences* (φ, ψ) of a propositional language.
- Belief sets (K) are modeled as *sets of sentences closed* under logical implication (theories).
- The revision of K by φ ($K * \varphi$) is modeled as a *function*, mapping theories and sentences to theories.



THE AGM POSTULATES FOR REVISION

- *Rational* revision functions, the so-called *AGM revision functions*, are constrained by *eight postulates*.
- They do not *uniquely* specify the new belief set $K * \varphi$.
- They simply circumscribe the territory of all different *rational* ways of revising belief sets.



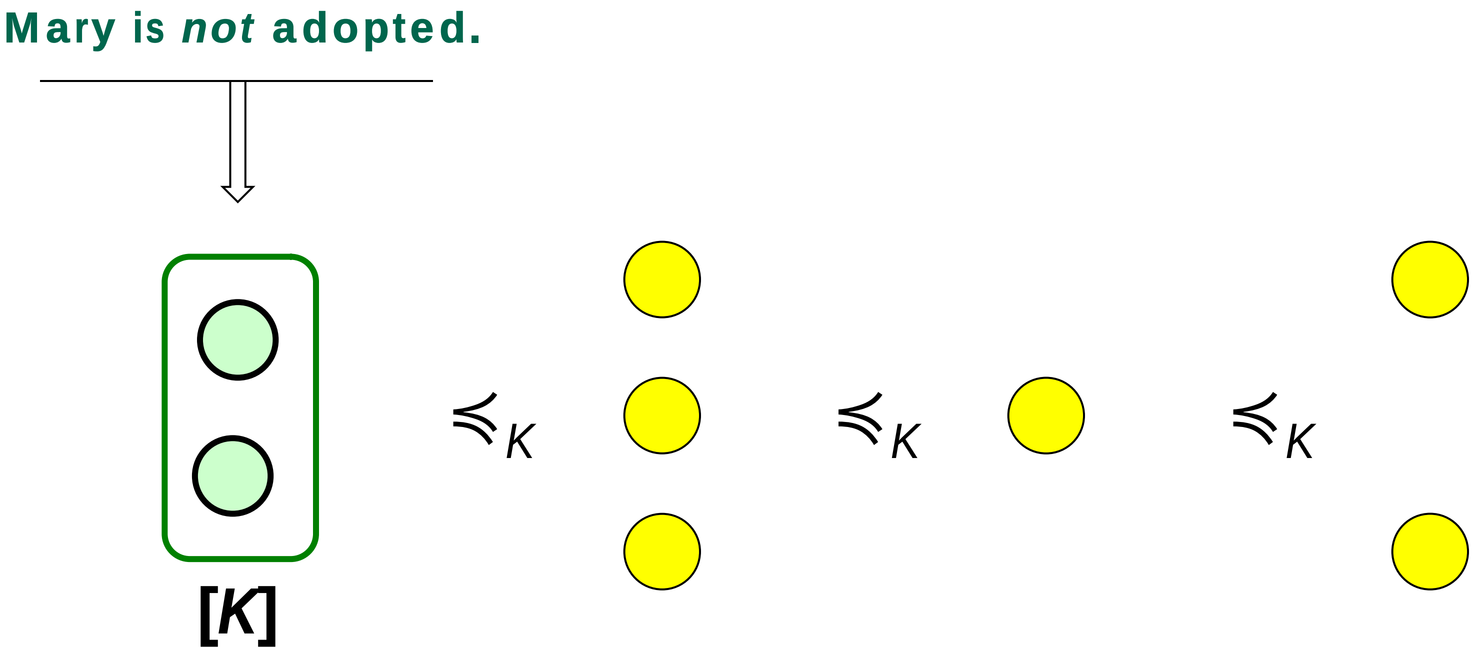
AGM Revision Functions

- We need *constructive models* for belief revision.

Faithful Preorders

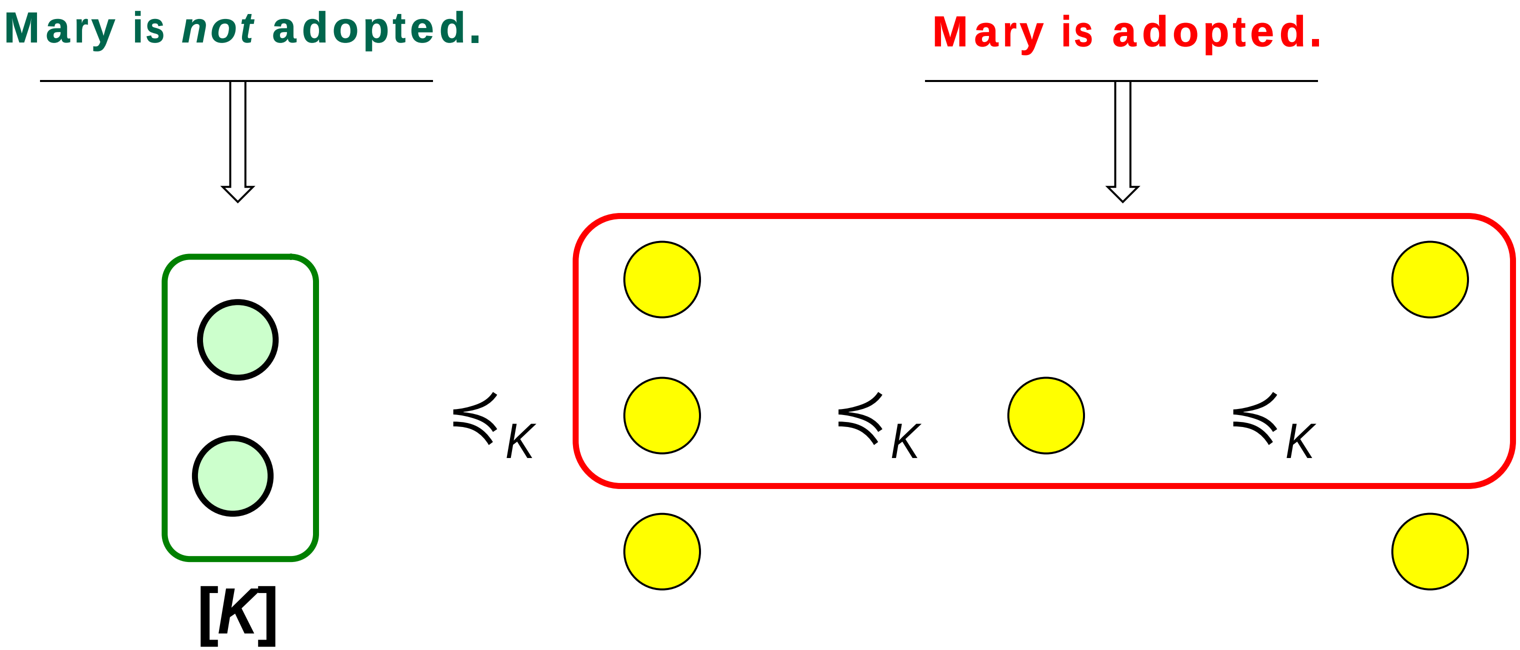
- There are many ways to *construct* an AGM revision function, but they are all equivalent to specifying a total preorder over possible worlds — called *faithful preorder* and denoted by $\preccurlyeq_K$ — for *every* theory K of the language.
- Recall that a *possible world* (or simply a world) is a *maximal* consistent subset of the underlying language.
- In every possible world, *each* sentence of the language is either true or false.

FAITHFUL PREORDERS – AN EXAMPLE [1/3]

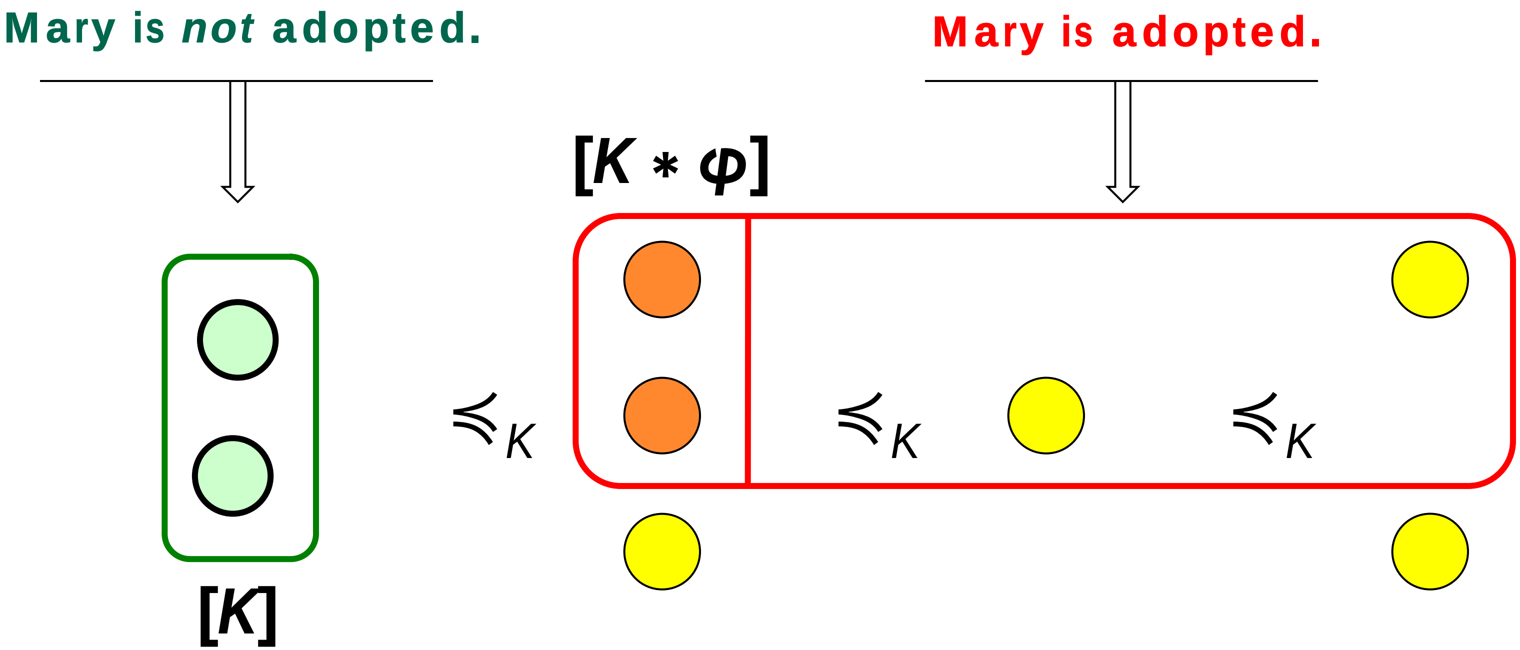


The preorder $\preccurlyeq_K$ represents a plausibility ranking over worlds, with respect to K; the more plausible a world is, the lower it appears in the ranking.

FAITHFUL PREORDERS – AN EXAMPLE [2/3]



Faithful Preorders – An Example [3/3]

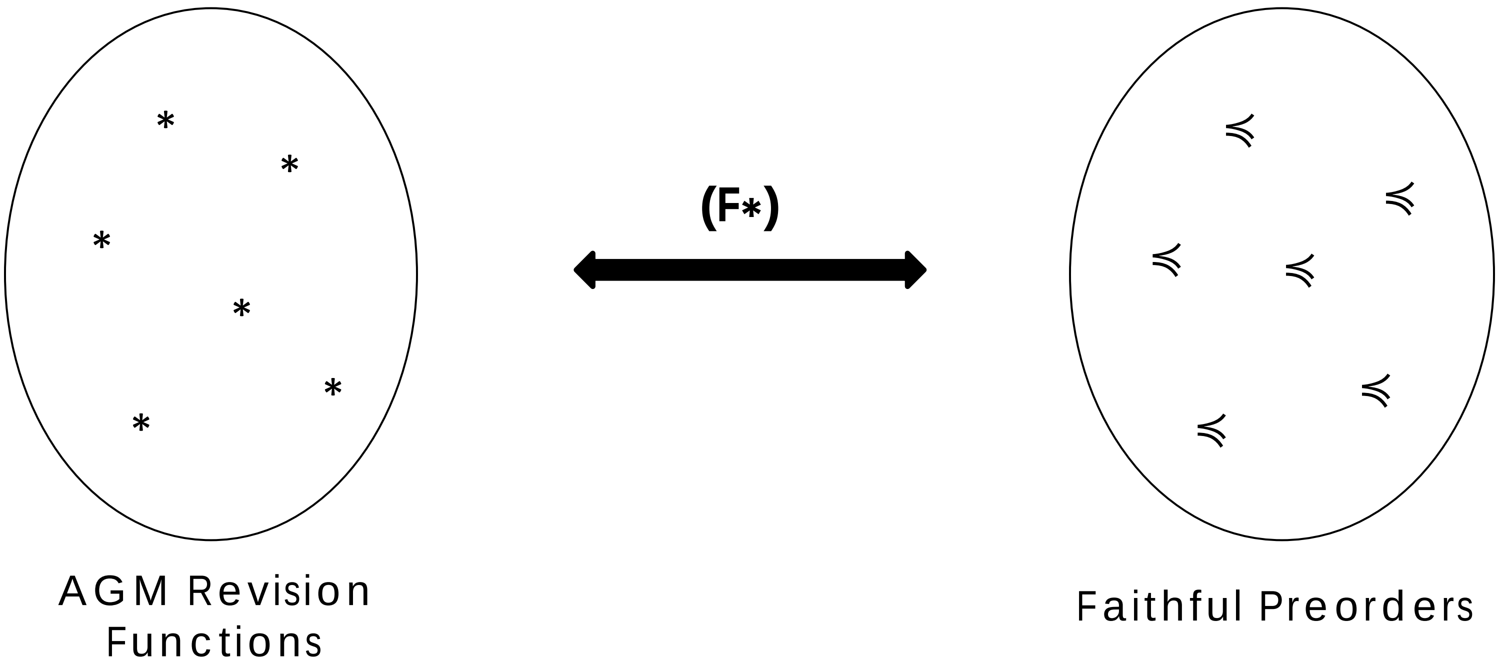


$$[K * \varphi] = min([\varphi], \preccurlyeq_K).$$

$K * \varphi$ is the theory corresponding to the most plausible φ-worlds.

Representation Result

The family of functions constructed from faithful preorders, by means of **(F∗)**, is precisely the class of AGM revision functions.



$$\textbf{(F}*\textbf{)} \qquad [K * \varphi] = \mathit{min}([\varphi], \preccurlyeq_K).$$

Parikh's Notion of Relevance

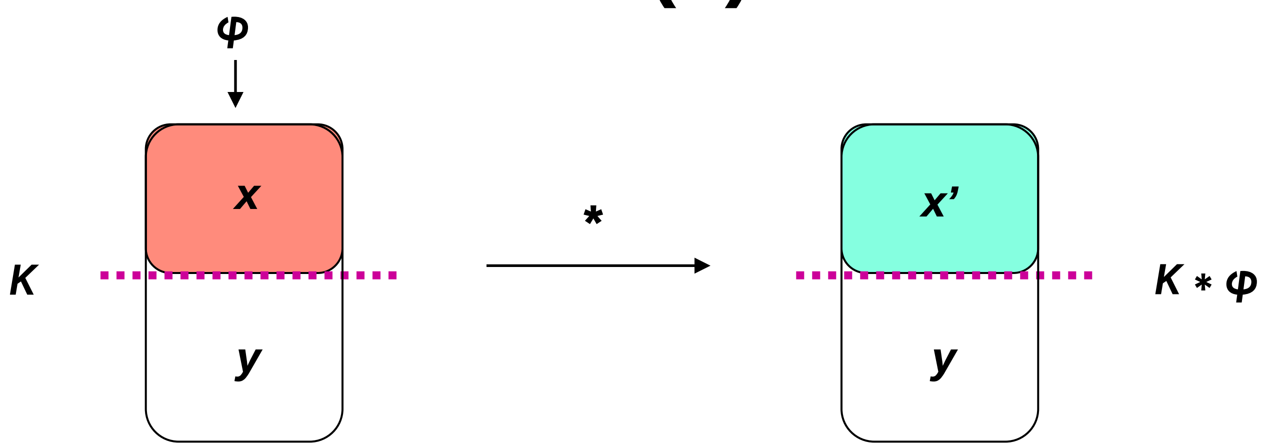
- When revising a theory K by a sentence φ, only the beliefs that are *relevant* to φ should be affected, while the rest of the belief corpus remains *unchanged*.

- This simple intuition is not fully captured by the AGM paradigm.

- For this reason, Parikh introduced a new axiom, named (P), as a supplement to the AGM postulates.

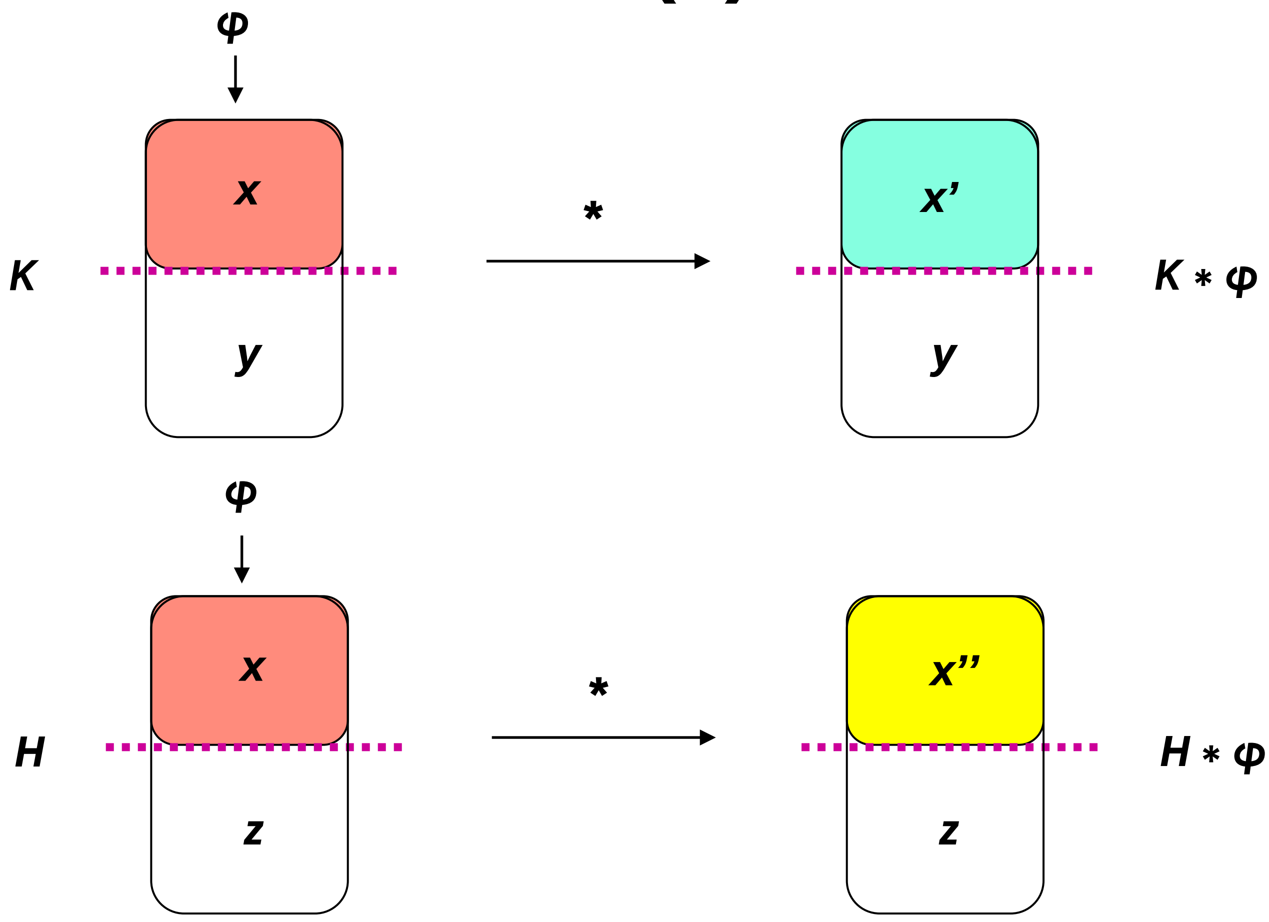
- Axiom (P) is open to two different interpretations; i.e., the *weak* and the *strong* version of (P).

WEAK (P)



(P1) If $K = Cn(x, y)$, $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$, and $\varphi \in \mathcal{L}_x$, then $(K * \varphi) \cap \overline{\mathcal{L}_x} = K \cap \overline{\mathcal{L}_x}$

WEAK (P)



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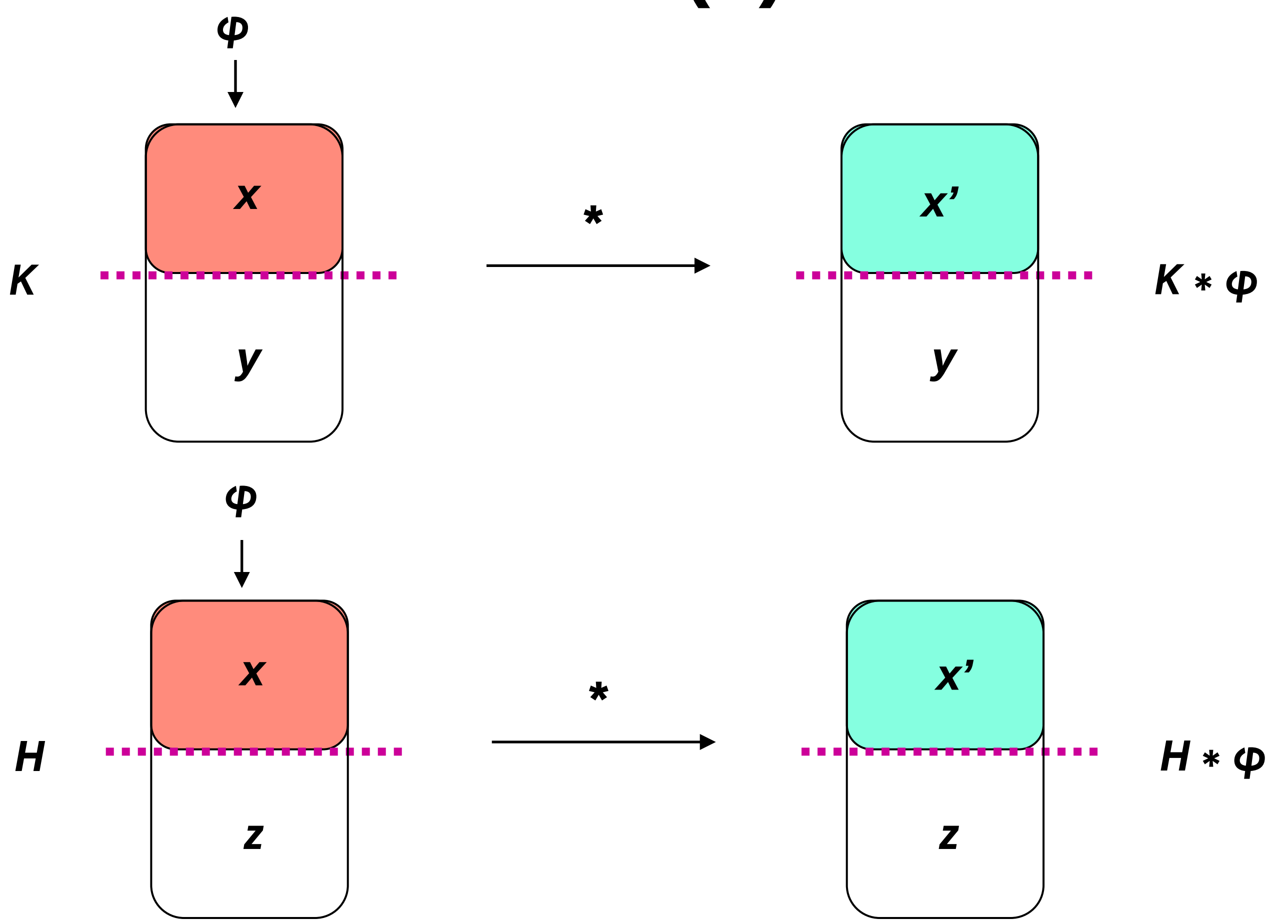
STRONG (P)



(P1) If $K = Cn(x, y)$, $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$, and $\varphi \in \mathcal{L}_x$, then $(K * \varphi) \cap \overline{\mathcal{L}_x} = K \cap \overline{\mathcal{L}_x}$

(P2) If $K = Cn(x, y)$, $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$, and $\varphi \in \mathcal{L}_x$, then $(K * \varphi) \cap \mathcal{L}_x = (Cn(x) * \varphi) \cap \mathcal{L}_x$

Faithful-Preorders Characterization of (P1)

- The faithful-preorders characterization of (P1) — that is, *weak* (P) — has been formulated in terms of a notion of *difference* between possible worlds (i.e., *Diff*(*r*,*r'*)), and between theories and possible worlds (i.e., *Diff*(*K*,*r*)).

(Q1) If $\mathit{Diff}\,(K, r) \subset \mathit{Diff}\,(K, r')$ and $\mathit{Diff}\,(r, r') \cap \mathit{Diff}\,(K, r) = \varnothing$, then $r \prec_K r'$

(Q2) If $\mathit{Diff}\,(K, r) = \mathit{Diff}\,(K, r')$ and $\mathit{Diff}\,(r, r') \cap \mathit{Diff}\,(K, r) = \varnothing$, then $r \approx_K r'$

- Whenever the agent, who holds a theory *K*, arranges the possible worlds according to the dictates of (Q1)–(Q2), the revision functions induced satisfy weak (P).

Faithful-Preorders Characterization of (P2)

- Let $K = Cn(x, y)$, such that, for some $x, y \in \mathcal{L}$, $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$.

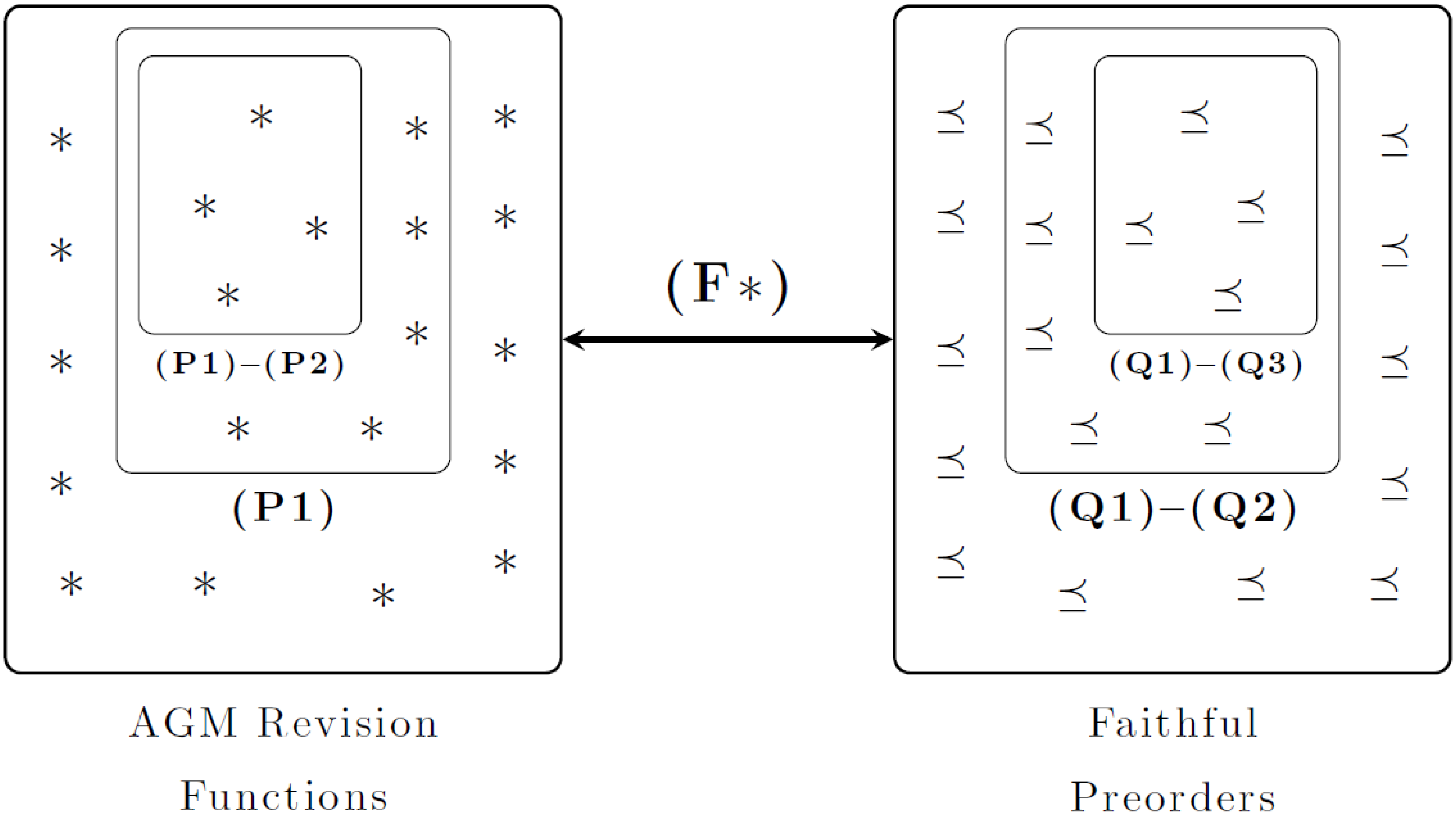
- For a faithful preorder $\preccurlyeq_K$, the ***x*-filtering** of $\preccurlyeq_K$, denoted by $\preceq_K^x$, is defined as follows:

$$r \preceq_K^x r' \quad \textit{iff} \quad \textit{there is a world } w \in [r_x], \textit{ such that, for all } w' \in [r'_x],\ w \preceq_K w'$$

- The *x*-filtering is, essentially, a "*projection*" of the initial preorder to the minimal language of the sentence *x*.

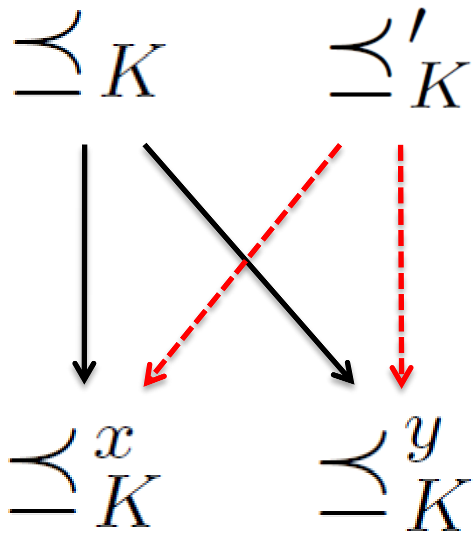
$$\textbf{(Q3)} \quad \text{If } K = Cn(x, y) \text{ and } \mathcal{L}_x \cap \mathcal{L}_y = \varnothing, \text{ then } \preceq_K^x = \preceq_{Cn(x)}^x$$

Faithful-Preorders Characterization of (P)



A Useful Remark for Filterings

Remark 1. *Let K be a splittable theory of $\mathcal{L}$, such that, for some contingent sentences $x, y \in \mathcal{L}$, $K = Cn(x, y)$ and $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$. Moreover, let $\preceq_K$ be a faithful preorder associated with K. Then, one can* uniquely *determine (via Definition 6) its filterings $\preceq_K^x$ and $\preceq_K^y$. The converse is not, in general, true; the preorders $\preceq_K^x$ and $\preceq_K^y$ cannot always* uniquely *determine the initial preorder $\preceq_K$, since there could be another preorder $\preceq'_K$, such that $\preceq'_K \neq \preceq_K$, $\preceq'^x_K = \preceq^x_K$ and $\preceq'^y_K = \preceq^y_K$.*



From Faithful Preorders to Their Filterings

Given a theory K of $\mathcal{L}$, if $\preccurlyeq_K$ satisfies conditions (Q1)–(Q2), then the (unique) filtering of $\preccurlyeq_K$, with respect to the sublanguage corresponding to *any* compartment of K, satisfies (Q1)–(Q2) as well.

Theorem 1. *Let K be a theory of $\mathcal{L}$, such that, for some sentences $x, y \in \mathcal{L}$, $K = Cn(x, y)$ and $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$. If the preorder $\preceq_K$ satisfies conditions (Q1)–(Q2), then the x-filtering of $\preceq_K$, namely $\preceq_K^x$, satisfies conditions (Q1)–(Q2).*

$\preceq_K$ **: (Q1)–(Q2)**

↓

$\preceq_K^x$ **: (Q1)–(Q2)**

FROM FILTERINGS TO FAITHFUL PREORDERS [1/2]

- Consider a splittable theory K of $\mathcal{L}$, and a faithful preorder $\preccurlyeq_K$ that satisfies (Q1)–(Q2); hence, from Theorem 1, $\preceq_K^x$ and $\preceq_K^y$ satisfy (Q1)–(Q2) as well.

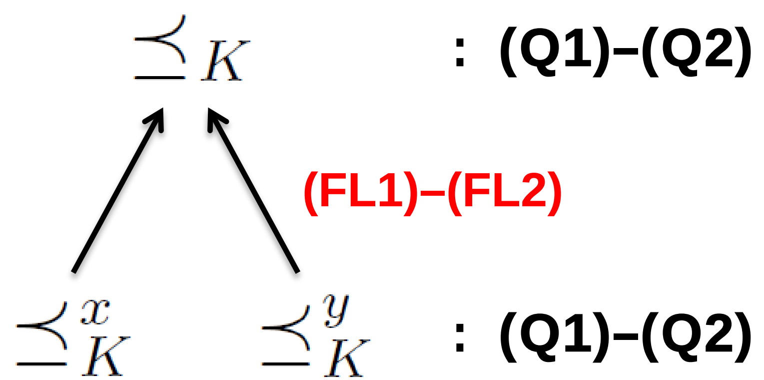
- In view of Remark 1, there could be another preorder $\preccurlyeq'_K$ such that $\preceq_K^{\prime x} = \preceq_K^x$ and $\preceq_K^{\prime y} = \preceq_K^y$. However, $\preccurlyeq'_K$ does not **necessarily** satisfy (Q1)–(Q2), although its filterings do satisfy (Q1)–(Q2).

- In order to define the class of preorders $\preccurlyeq_K$ that satisfy (Q1)–(Q2), given that $\preceq_K^x$ and $\preceq_K^y$ satisfy (Q1)–(Q2), conditions (FL1)–(FL2) are required.

From Filterings to Faithful Preorders [2/2]

(FL1) If $r \prec_K^x r'$ and $r \preceq_K^y r'$, then $r \prec_K r'$

(FL2) If $r \approx_K^x r'$ and $r \approx_K^y r'$, then $r \approx_K r'$



Economy of Resources Due to Strong (P) [1/2]

Proposition 2. *Let K be a theory of $\mathcal{L}$, such that, for some sentences $x, y \in \mathcal{L}$, $K = Cn(x,y)$ and $\mathcal{L}_x \cap \mathcal{L}_y = \varnothing$. Moreover, let $\preceq_{Cn(x)}$ be the faithful preorder associated with $Cn(x)$. If $\preceq_{Cn(x)}$ satisfies condition (Q2), then $\preceq_{Cn(x)}$ is identical to its x-filtering; in symbols, $\preceq_{Cn(x)} = \preceq^x_{Cn(x)}$.*

Suppose that an agent revises **any** theory K of $\mathcal{L}$ according to the dictates of strong (P). That is to say, the faithful preorder $\preccurlyeq_K$ that the agent holds satisfies conditions (Q1)–(Q3). Then, in view of Proposition 1, condition (Q3) is equivalent to condition (Q3)':

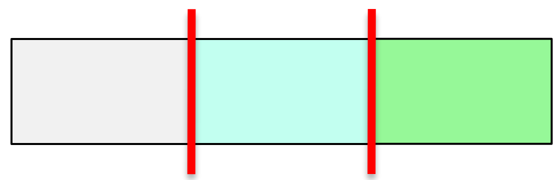
$$\textbf{(Q3)} \quad \text{If } K = Cn(x,y) \text{ and } \mathcal{L}_x \cap \mathcal{L}_y = \varnothing, \text{ then } \preceq^x_K = \preceq^x_{Cn(x)}$$

$$\textbf{(Q3)}' \quad \text{If } K = Cn(x,y) \text{ and } \mathcal{L}_x \cap \mathcal{L}_y = \varnothing, \text{ then } \preceq^x_K = \preceq_{Cn(x)}$$

Remark 2. *Suppose that a rational agent revises* any *theory of $\mathcal{L}$ according to the dictates of strong (P). Let K be a splittable theory of $\mathcal{L}$, such that, for some contingent sentences $x_1, \ldots, x_n \in \mathcal{L}$, $K = Cn(x_1, \ldots, x_n)$, and $\mathcal{L}_{x_i} \cap \mathcal{L}_{x_j} = \varnothing$, for all $1 \leqslant i \neq j \leqslant n$. Then, condition (Q3)′ implies that, whenever the agent holds the faithful preorder $\preceq_K$, she can* uniquely *determine the faithful preorders associated with all* $2^n - 2$ compartments *of K; i.e., she can uniquely determine the preorders $\preceq_{Cn(x_1)}, \preceq_{Cn(x_2)}, \ldots, \preceq_{Cn(x_1,x_2)}, \preceq_{Cn(x_1,x_3)}, \ldots$.*

Economy of Resources Due to Strong (P) [2/2]

Example 2. *Let* $\mathcal{P} = \{a, b, c\}$ *and* $K = Cn(a, b, c)$. *Clearly, theory* K *is splittable. Given strong (P), the faithful preorder* $\preceq_K$ *uniquely determines the faithful preorders associated with the following* $2^3 - 2 = 6$ *theories of* $\mathcal{L}$*:* $Cn(a)$, $Cn(b)$, $Cn(c)$, $Cn(a, b)$, $Cn(a, c)$, *and* $Cn(b, c)$. *For instance,* $\preceq_{Cn(a)} = \preceq_K^a$ *and* $\preceq_{Cn(a,b)} = \preceq_K^{a \wedge b}$.



***K* = *Cn* (*a*,*b*,*c*)**

For constructing an AGM revision function (encoding a revision policy), the agent needs a faithful preorder for **every** theory.

Remark 2 points out that, whenever the agent holds a faithful preorder for a **splittable** theory, strong (P) results in an **exponential drop** on the resources required.

CONCLUSIONS

- In this work, the strong version of Parikh's relevance-sensitive axiom (P) was further analyzed, based on previous work.
- Firstly, interesting features of faithful-preorder filterings were pointed out.
- Moreover, the economy of resources (in particular, an exponential drop) that strong (P) potentially results, for the construction of an AGM revision function, was highlighted.
- Given that the notion of relevance constitutes a cornerstone in many Artificial Intelligence domains, the established results are of interest in a plethora of applications.

Thank you!