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Should mention combinatorial principles derived from Cohen reals, notably in the work of Juhász, Soukup and Szentmiklósy (1998).

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Theorem

(D ž. 2015 Top. Appl) Cohen forcing adds a Boolean algebra $\mathfrak{A}$ of size $\aleph_1$ such that in the extension, $C(\mathrm{St}(\mathfrak{A}))$ does not embed into any Banach space which has a dense set of size $\aleph_1$ from the ground model. Moreover, the construction can be arranged so that $C(\mathrm{St}(\mathfrak{A}))$ is weakly compactly generated.

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Morass

A *simplified* $(\omega, 1)$*-morass* is a system $\langle \theta_\alpha : \alpha \leq \omega \rangle, \langle \mathfrak{F}_{\alpha,\beta} : \alpha < \beta \leq \omega \rangle$ such that

1. for $\alpha < \omega$, θ_α is a finite number > 0, and $\theta_\omega = \omega_1$,
2. for $\alpha < \beta < \omega$, $\mathfrak{F}_{\alpha,\beta}$ is a finite set of order preserving functions from θ_α to θ_β,
3. $\mathfrak{F}_{\alpha,\omega}$ is a set of order preserving functions from θ_α to ω_1 such that $\bigcup_{f \in \mathfrak{F}_{\alpha,\omega}} f``\theta_\alpha = \omega_1$,
4. for all $\alpha < \beta < \gamma \leq \omega$ we have that $\mathfrak{F}_{\alpha,\gamma} = \{f \circ g : g \in \mathfrak{F}_{\alpha,\beta} \text{ and } f \in \mathfrak{F}_{\beta,\gamma}\}$,
5. $\mathfrak{F}_{\alpha,\alpha+1}$ always contains the identity function id_α on θ_α and either this is all, or $\mathfrak{F}_{\alpha,\alpha+1} = \{\mathrm{id}_\alpha, h_\alpha\}$ for some h_α such that there is a *splitting point* β with $h_\alpha | \beta = \mathrm{id}_\alpha | \beta$ and $h_\alpha(\beta) > \theta_\alpha$,
6. for every $\beta_0, \beta_1 < \omega$ and $f_l \in \mathfrak{F}_{\beta_l,\omega}$ for $l < 2$ there is $\gamma < \omega$ with $\beta_0, \beta_1 < \gamma$, function $g \in \mathfrak{F}_{\gamma,\omega}$ and $f'_l \in \mathfrak{F}_{\beta_l,\gamma}$ such that $f_l = g \circ f'_l$ for $l < 2$.

Existence of morasses

Velleman (1984) proved that a simplified $(\omega, 1)$-morass exists in ZFC.

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(i) if $\beta < \alpha$ and $f \in \mathfrak{F}_{\beta,\alpha}$ then f gives rise to a Boolean algebra embedding.

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This guarantees that $\mathfrak{A}$ is indeed a limit of a direct system of Boolean algebras.

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$$||z_{j(i_0)} + z_{j(i_1)} + \ldots z_{j(i_{n_*^2})}|| < n_* - 1 \implies i_0, i_1, \ldots i_{n_*^2} \text{ disjoint in } \mathfrak{A} \text{ and}$$

$$||z_{j(i_0)} + z_{j(i_1)} + \ldots z_{j(i_{n_*^2})}|| \geq n_* - 1 \implies i_0 <_{\mathfrak{A}} i_1 \ldots <_{\mathfrak{A}} i_{n_*^2}. \quad (*)$$

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How to assure it

For any fixed X_* and $\{z_i : i < \omega_1\}$, it suffices to assure that for all

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there are $i_0, i_1 < \ldots < i_{n_*^2} \in A$ exemplifying $(*)$.

Our second requirement of the induction will be as follows, where r is the generic Cohen real, viewed as a function from ω to 2 :

(ii) Let $n < \omega$ and suppose that $\mathfrak{A}_{\alpha_n}$ has been defined. Then $\mathfrak{A}_{\alpha_{n+1}}$ is generated freely over $\mathfrak{A}_{\alpha_n}$ except for the equations induced by requirement **(i)** and the requirement that $i_0, i_1, \dots i_n$ are disjoint in $\mathfrak{A}_{\alpha_{n+1}}$ if $r(n) = 0$ and $i_0 <_{\mathfrak{A}_{\alpha_{n+1}}} i_1 <_{\mathfrak{A}_{\alpha_{n+1}}} \dots <_{\mathfrak{A}_{\alpha_{n+1}}} i_n$ if $r(n) = 1$, where $\{i_0, i_1, \dots, i_n\}$ is the increasing enumeration of the first $n+1$ elements of A_n. For $\beta \in (\alpha_n, \alpha_{n+1})$ we let $\mathfrak{A}_\beta$ be the Boolean span in $\mathfrak{A}_{\alpha_{n+1}}$ of θ_β.

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