Effective Properties of Abstract Structures

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Classical Descriptive Set Theory (DST) is built on the foundations provided by the topological notion of Polish space. It has been very successful with the analysis of metrizable spaces - it provides notions of definability, hierarchies and reducibilities. Among other things, DST can be viewed as a natural generalization of classical computability theory, e.g. continuous instead of Turing computable functions, Borel hierarchy instead of hyperarithmetical hierarchy, etc.

Recently, there has been some interest (e.g. [4], [2]) in developing DST for quasi-Polish spaces, e.g. Scott topology. This is not an easy task since these spaces are not Hausdorff. In this talk, my point of view will be that of a computability theorist and I will argue that DST on quasi-Polish spaces is the natural generalization of classical computability theory using only positive information about the underlying objects. This is the study of enumeration operators on $\mathcal{P}(\omega)$ instead of Turing operators on 2^ω. My main examples will be from the area of computable structure theory [1] where these ideas are applied to obtain new results about definability and reducibility of classes of structures similar to that of Friedman and Stanley [3].

References

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[4] V. Selivanov, *Towards a Descriptive Set Theory for Domain-like Structures*, Theoretical Computer Science, **365**, no. 3 (2006), pp. 258–282.