A Framework for Representing Views about Logic

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Abstract

This paper presents a framework for representing views about logic, as opposed to mere logic systems. A theory of logic is characterised as a collection of statements specifying which logic systems and topics exist, and which systems are correctly applicable to which topics. To formalise this, I introduce the notion of a system of applications, a structure on which theories of logic are based. This framework allows for a unified treatment of several views about logic, including logical monistic and pluralistic ones.

Introduction

A theory of logic is an account of how logical consequence works. Accounts on logical consequence are usually formalised via logic systems which are supposed to apply universally. However, a theory of logic is not just a set of formal rules of inference (cf. [9, 252–3], [10, 635]), but a statement about their truth or correctness. This paper presents a framework for representing theories and views about logic in this sense.

I will proceed as follows. Section 1 defines the concept of logic system and argues that it does not suffice for characterising theories of logic. Section 2 presents system of applications, on which theories of logic are based. We define the latter in section 3 as collections of statements about logic systems, logical topics, and their relationships. Section 4 defines the more general notion of *view about logic*, as a collection of theories of logic. Finally, section 5 defines some notable views about logic, including monism and pluralism.

Notation. The symbol '✠' marks the end of an enumerated statement (like a definition) – just like '■' usually marks the end of a proof. Other conventions are understandable in context.

1 Systems and Theories of Logic

Conventionally, logic systems are conceived as pairs consisting of a *formal language* and a *relation of logical consequence* – or *consequence relation* for short.

Definition 1.1. A *logic system* is a pair $\mathsf{S} = \langle \mathcal{F}_\mathsf{S}, \mathsf{Cons}_\mathsf{S} \rangle$, where $\mathcal{F}_\mathsf{S}$ is a set of formulae and $\mathsf{Cons}_\mathsf{S} \subseteq 2^{\mathcal{F}_\mathsf{S}} \times \mathcal{F}_\mathsf{S}$ is a consequence relation. ✠

Thus, stating that $(\mathcal{A}, \mathtt{A}) \in \mathsf{Cons}_\mathsf{S}$ amounts to state that $\mathtt{A} \in \mathcal{F}_\mathsf{S}$ is an S-consequence of the formulae in $\mathcal{A} \subseteq \mathcal{F}_\mathsf{S}$ – i.e., it follows from $\mathcal{A}$ *logically*, in the sense formalised by S.

Under logical monism – the view that there is exactly one correct logic system – a definition of a theory of logic just requires specifying some logic system as the *one* which is correctly applicable in any context in which logic can be conceivably applied. Therefore, for the logical monist, the concept of a theory of logic may be safely reduced to that of a system of logic.

Logical pluralism – the view that there is more than one correct logic system – challenges this account a bit. A logical pluralist often assumes the thesis that *topic-neutralism*, the thesis that any logic system which is considered correct must be correctly applicable to any logical topic, that is, to any topic in which logic can be conceivably applied[1]. For one such logical pluralist, a theory of logic would only need to indicate the set of those logic systems which are correctly applicable indifferently of the logical topic Thus, for this logical pluralist, the concept of a theory of logic may be reduced to that of a set of logic systems.

The real challenge arises from questioning *topic-neutralism*. This is no minor revision of standard assumptions about logic, since many authors take topic-neutrality to be a defining feature of logic (cf. [8, 16]), There are nevertheless other authors who have argued that some logic systems might be appropriate for dealing with some topics but inappropriate for others. This has resulted in views such as logical contextualism [1] and logical modalism [4, 3], the latter explicitly defending that logic is topic-specific.

Rejecting topic-neutralism might seem beyond acceptable to some. However, in order to make sense of all views about logic, we need incorporate logical topics as part of our characterisation of them, as well as how they relate to logic systems in terms of correct applicability.

2 Systems of Applications

In very simple terms, a theory of logic states: (a) which mathematical systems are logic system; (b) what things qualify as logical topics (i.e., those topics about which logic is topic-neutral or not); (c) how those logic systems above are correctly applicable or not to these topics.

In order to define a notion of a theory of logic with those features, I will first introduce the notion of a *system of applications*.

Definition 2.1. A *system of applications* is a triple $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$ with $\mathbf{C} \subseteq \mathscr{T} \times \mathscr{S}$, where:

- $\mathscr{T} = \{\mathbb{T}_1, ..., \mathbb{T}_l\}$ is a non-empty set of topics; and
- $\mathscr{S} = \{\mathsf{S}_1, ..., \mathsf{S}_m\}$ is a non-empty set of logic systems. ✠

We introduce the following notation for the image and pre-image of subsets of $\mathscr{T}$ and $\mathscr{S}$:

Definition 2.2. Let $\mathbf{C} \subseteq \mathscr{T} \times \mathscr{S}$. The *pre-image* and *image* of $\mathscr{T}' \subseteq \mathscr{T}$ and $\mathscr{S}' \subseteq \mathscr{S}$ under $\mathbf{C}$ are defined as follows:

$$\mathbf{Pr_C}(\mathscr{S}') = \{\mathbb{T} \in \mathscr{T} : (\mathbb{T}, \mathsf{S}) \in \mathbf{C}, \text{ for some } \mathsf{S} \in \mathscr{S}'\},$$
$$\mathbf{Im_C}(\mathscr{T}') = \{\mathsf{S} \in \mathscr{S} : (\mathbb{T}, \mathsf{S}) \in \mathbf{C}, \text{ for some } \mathbb{T} \in \mathscr{T}'\}.$$ ✠

In words, $\mathbf{Pr_C}(\mathscr{S}')$ is the set of logical topics mapped to some logic system in $\mathscr{S}'$ by $\mathbf{C}$, and $\mathbf{Im_C}(\mathscr{T}')$ is the set of logic systems mapped to some logical topic in $\mathscr{T}'$ by $\mathbf{C}$.

We will employ the following abbreviations for $\mathbf{C} \subseteq \mathscr{T} \times \mathscr{S}, \mathbb{T} \in \mathscr{T}, \mathsf{S} \in \mathscr{S}$:

$$\mathbf{Pr_C} := \mathbf{Pr_C}(\mathscr{S}), \quad \mathbf{Im_C} := \mathbf{Im_C}(\mathscr{T}), \quad \mathbf{Pr_C}(\mathsf{S}) := \mathbf{Pr_C}(\{\mathsf{S}\}), \quad \mathbf{Im_C}(\mathbb{T}) := \mathbf{Pr_C}(\{\mathbb{T}\}).$$

That is, $\mathbf{Pr_C}$ is the set of logical topics mapped to some logic system by $\mathbf{C}$ and $\mathbf{Im_C}$ is the set of logic systems mapped to some logical topic by $\mathbf{C}$.

[1] A definition of the concept of a *logical topic* is here lacking, and it will left for a future work. For a detailed account of the concept of *topic* that is relevant to my notion of a logical topic, see Berto [2]. Similarly, the sense of *correctly applicable* will be left vague here, although it corresponds to the sense in which a given logical system is said to be *the true or correct logic* in the relevant literature.

3 Theories of Logic

Just as a logic system, a system of applications alone will not suffice for expressing a view about logic. It will be in the context of a *complete theory of logic* – or *theory of logic*, for short – that a system of applications may serve to express it.

Definition 3.1. A (*complete*) *theory of logic* is a collection of statements stating: (i) exactly which logical topics exist; (ii) exactly which mathematical systems qualify as logical systems; (iii) exactly which logical systems are correctly applicable to which logical topics. ✠

The way to relate systems of applications to theories of logic is by defining a way to construct a theory of logic from a given system of applications. That is, we want the notion of a theory *being based on* a system of applications.

Definition 3.2. A theory of logic *is based on* a system of applications $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$ iff the theory's statements are exactly as follows:

- $\mathbb{T}$ is a topic iff $\mathbb{T} \in \mathscr{T}$.
- S is a logic system iff $\mathsf{S} \in \mathscr{S}$.
- S is correctly applicable to $\mathbb{T}$ iff $\langle \mathbb{T}, \mathsf{S} \rangle \in \mathbf{C}$. ✠

We can now define a theory just by specifying the system of applications it is based on.

Note that, in the context of a theory of logic, it may happen that $\mathbf{Im}_{\mathbf{C}} \subset \mathscr{S}$ – that is, the theory may postulate the existence of logic systems which are not correctly applicable to any logical topic. This is not problematic. In practice, it is often difficult to specify necessary and sufficient conditions for logical systems, even when we have strong intuitions about which systems should or should not count as correctly applicable to a given logical topic.

Let us give an example of how to define a theory of logic in this framework. Consider a theory stating that classical logic is the only logic system that is correctly applicable to the most general topic. Call this theory *generalistic classicalism.* We may define it as follows:

Definition 3.3. *Generalistic classicalism* is the theory (of logic) based the a system (of applications) $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, where:

- $\mathscr{T} = \{\mathbb{G}\}$, where $\mathbb{G}$ is the most general topic;
- $\mathscr{S} = \{\mathsf{C}\}$, where C is classical logic; and
- $\mathbf{C} = \{\langle \mathbb{T}, \mathsf{C} \rangle\}$. ✠

In other words, generalistic classicalism states the following:

- There is only one logical topic, which is the most general topic.
- There is only one logic system, which is classical logic (or C).
- Classical logic is correctly applicable to the most general topic.

Note that only the elements of $\mathscr{T}$ and $\mathscr{S}$ denote logic systems and logical topics according to generalistic classicalism. Whatever is outside these sets fails to denote logic systems and logical topics according to this theory. Similarly, only the pair $\langle \mathbb{G}, \mathsf{C} \rangle \in \mathbf{C}$ is related in terms of correct applicability according to this theory. Any pair $\langle \mathbb{T}, \mathsf{S} \rangle \notin \mathbf{C}$ is such that S is not correctly applicable to $\mathbb{S}$ according to this theory. It is in this sense that a theory of logic is *complete*.

Generalistic classicalism is likely the received theory of logic.

4 Views about Logic

A view about logic is seldom advanced as a complete theory of logic, understood in the sense above. Rather, it typically consists of a set of claims that could be met by various – often infinitely many – such theories.

For instance, logical pluralism simply states that there is more than one correctly applicable logic system. There are nevertheless numerous sets of logic systems which we can choose and several ways in which we can relate them to logical topics in terms of correct applicability. There is, however, a straightforward way to define such views about logic within our framework.

Definition 4.1. A *view about logic* is a non-empty class of theories of logic. ✠

Consider, for instance, a view about logic that I will refer to as *anti-classicalism*, according to which classical logic is not correctly applicable to any logical topic whatsoever.

Definition 4.2. *Anti-classicalism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, where:

- $\mathsf{C} \in \mathscr{S}$, where C is classical logic; and
- for all $\mathbb{T} \in \mathscr{T}$, we have $\langle \mathbb{T}, \mathsf{C} \rangle \notin \mathbf{C}$. ✠

In other words, an *anti-classicalist* theory states the following:

- Classical logic (C) is a logic system.
- There is at least one logical topic (because a system of applications $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$ is such that $\mathscr{T} \neq \{\} \neq \mathscr{S}$).
- Classical logic is not correctly applicable to $\mathbb{T}$.

This is obviously compatible with many theories, since it says nothing about which logic systems are correctly applicable to which logical topics. Thus, an anti-classicalist view does not answer: whether C is the only logic system; exactly how many and which logical topics are there; nor how these additional systems may relate to the logical topics. Theories qualifying as anti-classicalist may answer these questions in different ways.

Anti-classicalism is not a widely held view, as most authors grant classical logic a role in at least some domain. An exception might be Nye [11].

5 Notable Views about Logic

I will now reconstruct some notable views about logic within this framework.

The first such view *pluralism*, the view that there is more than one correct or true logic. I construe it, in this framework, as the view asserting that there are at least two logic systems such that each is correctly applicable to at least one logical topic.

Definition 5.1. *Pluralism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, such that $\mathbf{Im}_{\mathbf{C}}$ has at least two members. ✠

The next view is *monism*, the view that there is only one true or correct logic. I construe it as the view asserting that there is exactly one logic system correctly applicable to all logical topics to which at least one logic system is correctly applicable.

Definition 5.2. *Monism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, such that $\mathbf{Im_C}$ is a singleton. ✠

Next, we have *nihilism*, the view that there is no correct logic. I construe it as the view asserting that there is no logical topic to which a logic system is correctly applicable.

Definition 5.3. *Nihilism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, such that $\mathbf{Im_C} = \{\}$. ✠

In other words, logic is correctly applicable nowhere. Although this definition of logical nihilism appears natural, it does not reflect how this view is conceived in the literature (cf. [14]). Instead, standard nihilism appears to be a variety of topic-specificism (see definition 5.6)

Next, *universalism*, the view that logic is universal. I construe it as the view asserting that, for any logical topic, there is at least one logic system which is correctly applicable to it.

Definition 5.4. *Universalism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, such that, for all $\mathbb{T} \in \mathscr{T}$, we have $\mathbf{Im_C}(\mathbb{T}) \neq \{\}$. ✠

In other words, there is no logical topic to which no logic system is correctly applicable.

Next, we have *topic-neutralism*, the view that logic is topic-neutral. I construe it as the view asserting that: (i) there is at least one logic system correctly applicable to at least one logical topic; and, (ii) for any two logical topics having at least one logic system correctly applicable to them, the sets of all logic systems correctly applicable to them are identical.

Definition 5.5. *Topic-neutralism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, such that: (i) $\mathbf{C} \neq \{\}$ and (ii) for all $\mathbb{T}, \mathbb{U} \in \mathbf{Im_C}$, we have $\mathbf{Im_C}(\mathsf{T}) = \mathbf{Im_C}(\mathsf{U})$. ✠

In other words, except for those logical topics to which no logic system is correctly applicable, logical topics are equal before logic.

Finally, *topic-specificism*, the view that logic is topic-specific. I construe it as asserting that: (i) there is at least one logic system correctly applicable to at least one logical topic; and (ii) there are at least two logical topics with different logic systems correctly applicable to them.

Definition 5.6. *Topic-specificism* is the view comprising all theories based on a system $\mathfrak{A} = \langle \mathscr{T}, \mathscr{S}, \mathbf{C} \rangle$, such that: (i) $\mathbf{C} \neq \{\}$ and (ii) there are $\mathbb{T}, \mathbb{U} \in \mathbf{Im_C}$ with $\mathbf{Im_C}(\mathbb{T}) \neq \mathbf{Im_C}(\mathbb{U})$. ✠

In other words, even excluding those logical topics to which no logic system applies, logical topics are not equal before logic.

In what follows, we will call a view about logic is *v-istic* iff it is a subclass of *v-ism*; similarly, a theory of logic will be called *v-istic* iff it falls within *v*-ism; otherwise, they will be called *non-v-istic*. For instance, a view or a theory about logic is *nihilistic* iff it is a subclass of or falls within *nihilism*, respectively; otherwise, they are *non-nihilistic*.

We will finish stating a couple of corollaries following from these definitions when we restrict the domain to non-nihilistic theories.

Corollary 5.7. A non-nihilistic theory of logic is topic-specific iff it is not topic-neutral. ✠

Corollary 5.8. A non-nihilistic theory of logic is pluralistic iff it is not monistic. ✠

6 Concluding Remarks

The framework presented in this paper provides a precise way to define views about logic. This approach, I believe, allows for a unified treatment of the most important views about logic, and for a better understanding of the relationships between them.

Moreover, the framework makes it possible to represent some sophisticated views about logic, like those by Bertrand Russell [13], who advocated a form of monistic non-universalism, or by Paulette Destouches-Février [7], who endorsed a form of topic-specific pluralism. At the same time, other positions – such as logical globalism, logical contextualism [1], logical relativism [15], systematisation [6], and various forms of logical pluralism advanced by Cook [5] or Priest [12] – may require further extensions of the framework in order to be represented.

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