On the Exponential Succinctness of Shannon-Uncertainty Modalities over Probabilistic Beliefs in Temporal Logic

Abstract

Epistemic agents have to face multiple layers of uncertainty. One source comes from agents' limited ability to observe their environment, while another arises from the unpredictability of natural events and the actions of other agents, which, though uncertain, can be estimated through experiments or past experiences. In this context, we shall look at one of the aspects of rationality – agents' ability to achieve their goals. These goals are often epistemic, involving the acquisition of partial or complete knowledge about a crucial fact A. Many such properties can be expressed using PATLK, an extension of probabilistic alternating-time temporal logic (PATL) with knowledge operators, or PATLC that extends PATL with probabilistic beliefs. In many scenarios, however, the goal of the players is not to achieve high confidence about A being true, but rather to reduce their uncertainty about A (be it true or false). Similarly, in scenarios where the goal is to keep A secret, the outsiders' uncertainty about A should be maintained above a certain threshold. To capture such properties, we introduce PATLH, a logic extending PATL with information-theoretic modalities based on Shannon Entropy. The logic enables the specification of agents' capabilities concerning the uncertainty of a player about a given set of facts. We define it over multi-agent systems with stochastic transitions and probabilistic imperfect information, capturing two key uncertainties: the agents' partial observability of their environment and the stochastic nature of state transitions.

In this paper, we prove that **PATLH** is exponentially more succinct than **PATLC**. In prior work it was shown that **PATLC** strictly subsumes **PATLH** in expressive power, yet it was conjectured that any **PATLC**-specifiable property could be encoded with exponentially shorter formulas than in **PATLH**. Establishing this remained elusive, since existing succinctness proofs (e.g., for non-probabilistic **ATLH** vs **ATLK**) do not readily transfer to the Shannon-entropy setting. Here we resolve the conjecture by explicitly constructing a family of **PATLH** formulas φ_n of size $O(n)$ and proving that any equivalent **PATLC** formula requires size $2^{\Omega(n)}$. Our argument uses the one-person formula-size-game framework. This formally confirms a conjecture from the previous **PATLH/PATLC** study, demonstrating a genuine exponential succinctness gap and advancing recent developments in logical succinctness for strategic-epistemic logics.

Contents

1 Introduction

Epistemic agents, especially in multi-agent systems often require reasoning about uncertain information and strategic behavior, blending probabilistic and epistemic considerations. In particular, probabilistic alternating-time temporal logic (**PATL**) and its extensions have been proposed to specify goals such as reaching knowledge or belief thresholds about facts under uncertainty. For example, **PATL** with knowledge operators (**PATLK**) can express that a coalition can ensure an agent comes to know a fact, while PATL with belief (credal) operators (**PATLC**) expresses that a coalition can ensure an agent's subjective probability exceeds a threshold. More recently, [7] introduced logic **PATLH**– an extension of **PATL** with Shannon-entropy modalities, to capture goals about reducing agents' uncertainty.

These logics differ in how concisely they can express specifications (for the ones they can both express). **PATLC** has strictly more expressive and distinguishing power than **PATLH** which means that formulas based on probabilistic knowledge allow to express a strictly larger class of properties than uncertainty-based ones which is very surprising, and makes probabilistic logics distinctly different from non probabilistic ones (where both kinds of modalities were equally expressive). On the other hand, the **PATLC** translations of uncertainty-based formulas are quite complicated and at least exponentially longer than its **PATLH** counterpart. As we show in this paper, this is no coincidence. We study this tradeoff in expressive succinctness: intuitively, one logic is *exponentially more succinct* than another if some properties require exponentially larger formulas when expressed in the less succinct language. Succinctness is important because it affects the practical usability of a logic for specification and verification. Our main contribution is to show that **PATLH** is *exponentially more succinct* than **PATLC**. In other words, there exists a family of properties that can be encoded by **PATLH** formulas of linear size, but any equivalent **PATLC** formula must have exponential size.

2 Preliminaries

Stochastic iCGS. *Stochastic Imperfect Information Concurrent Game Structures* (*Stochastic iCGS*, or *SiCGS*) extend the well-known Concurrent Game Structures [2] to allow for both imperfect information and stochastic transitions. Formally, a Stochastic iCGS is a tuple $\mathcal{G} = (\mathbb{A}\mathrm{gt}, St, Act, d, \delta, \{\sim_a\}_{a\in\mathbb{A}\mathrm{gt}})$, where $\mathbb{A}\mathrm{gt}$ is a finite set of agents, St is a finite set of states, Act is a finite set of actions, $d : \mathbb{A}\mathrm{gt}\times St \to 2^{Act}\setminus\{\emptyset\}$ is a function defining the available actions for each agent in each state, $\delta : St\times Act^n \to \mathcal{D}(St)$ is a stochastic transition function (where $\mathcal{D}(St)$ denotes the set of probability distributions over St) that gives the (conditional) probability $\delta(s,\mathbf{c})$ of a transition from state s for all $s' \in St$ if each player $a \in \mathbb{A}\mathrm{gt}$ plays the action $\mathbf{c_a}$ (we also write this probability as $\delta(s,\mathbf{c})(\mathbf{s'})$ to emphasize that $\delta(s,\mathbf{c})$ is a probability distribution on St), and $\sim_a\subseteq St \times St$ for each $a \in \mathbb{A}\mathrm{gt}$ is an equivalence relation capturing the indistinguishability of states for agent a.

Strategies and Probabilistic Outcomes. For the interplay between a coalition $C \subseteq \mathbb{A}\mathrm{gt}$ and its opponents $\overline{C} = \mathbb{A}\mathrm{gt} \setminus C$, we follow [3] and assume that C play *uniform deterministic memoryless strategies* (ir-strategies in short), whereas $\overline{C}$ can respond with any pattern of behavior (possibly probabilistic and history-based). Formally, an *ir-strategy* for agent a is a function $\sigma_a : St \to Act$in which: (i) for each s, we have $\boldsymbol{\sigma}_a(s) \in d(s,a)$; and (ii) if $s \sim_a s'$ then $\boldsymbol{\sigma}_a(s) = \boldsymbol{\sigma}_a(s')$. Moreover, a *general strategy* for agent $a \in \mathbb{A}\mathrm{gt}$ is represented by $\boldsymbol{\sigma}_a : St^+ \to \mathcal{D}(Act)$ that maps each finite history to a probability distribution over the agent's actions, such that $\boldsymbol{\sigma}_a(h) \in d(last(h), a)$. A *collective strategy* $\boldsymbol{\sigma}_C$ for C is a tuple of strategies $\boldsymbol{\sigma}_a$, one per agent $a \in C$.

The *outcome* of strategy $\boldsymbol{\sigma}_C$ from state s is the set $out_C(\boldsymbol{\sigma}_C, s) = \{out((\boldsymbol{\sigma}_C, \boldsymbol{\sigma}_{\overline{C}}), s) \mid \boldsymbol{\sigma}_{\overline{C}} \in \Sigma_{\overline{C}}\}$ of probability distributions over infinite paths in the model, consistent with C's choices prescribed by $\boldsymbol{\sigma}_C$, where Σ denotes the set of all possible strategies. Each distribution $\mu_{\boldsymbol{\sigma}_C,s} \in out_C(\boldsymbol{\sigma}_C, s)$ is obtained from the Markov chain that combines the Stochastic iCGS M with $\boldsymbol{\sigma}_C$ and a possible general strategy of $\overline{C}$. We refer to [3] for a detailed construction.

Probabilistic Alternating-Time Logic PATL. We now introduce Probabilistic Alternating-Time The language of **PATL*** is defined as follows: $\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi\wedge\varphi \mid \varphi\,\mathcal{U}\,\varphi \mid \bigcirc\varphi \mid \langle\!\langle C\rangle\!\rangle^{\propto p}\varphi$, where

$\langle\langle C\rangle\rangle^{\propto p}\varphi$ intuitively means that there exists a strategy for the coalition C to collaboratively enforce φ with a probability in relation $\propto$ with constant p, where $\propto\in\{=,\neq,>,<,\geq,\leq\}$.

The language of **PATL** is a restriction of the language of **PATL*** and is defined by the following grammar: $\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi\wedge\varphi \mid \langle\langle C\rangle\rangle^{\propto p}\bigcirc\varphi \mid \langle\langle C\rangle\rangle^{\propto p}\varphi\,\mathcal{U}\,\varphi$.

An iSCGS together with a set of atomic propositions AP and a valuation function $V : AP \to 2^{St}$ is called a stochastic concurrent game model (SCGM). The formulas of **PATL** are interpreted over pairs (M,π), consisting of an SCGM and an infinite path in it.

Satisfaction relation. The semantics of **PATL** extends the semantics of **ATL** by the following clause: $M,\pi \models \langle\langle C\rangle\rangle^{\propto p}\varphi]$ iff there is an ir-strategy $\boldsymbol{\sigma}_C$ s.t. for all $\mu_{\boldsymbol{\sigma}_C,\pi_0} \in out_C(\boldsymbol{\sigma}_C,\pi_0)$ we have $\mu_{\boldsymbol{\sigma}_C,\pi_0}(\{\rho : M,\rho\models\varphi\}) \propto p$.

We define $M,s\models\varphi$ iff $M,\pi\models\varphi$ holds for any π starting in s. $Sat(M,\varphi)=\{s\in St \mid M,s\models\varphi\}$ is the subset of states in M satisfying φ. Moreover, *pointed models* are pairs (M,s) consisting of an SCGM and a state in it. For a subset of pointed models A, we use $A\models\varphi$ to mean that for all $(M,s)\in A$ it holds that $M,s\models\varphi$. We also use $\hat{M}$ to denote the set of all pointed models in M.

3 Adding Knowledge and Uncertainty

In this section we introduce probabilistic extensions of **ATLK** [12] and **ATLH** [11] that can be simultaneously seen as epistemic and information-theoretic extensions of **PATL** [4].

Alternating-time temporal epistemic logic **ATLK** adds the (full) knowledge modality of the *multi-agent epistemic logic* to **ATL** with imperfect information.

In multi-agent epistemic logic, *knowledge* of agents is formalized by epistemic formulas $\mathcal{K}_a\varphi$, stating "agent a knows that φ holds." They are interpreted by the following clause:

$M,s\models\mathcal{K}_a\varphi$ iff, for every state t such that $s\sim_a t$, we have that $M,t\models\varphi$,

where $\sim_a\subseteq St\times St$ is an *epistemic equivalence relation* connecting states that are indistinguishable to a.

Analogously, **PATL** can be extended by operators $K_a^{\propto q}$ for probabilistic knowledge/beliefs, or *credences* (where $\propto\in\{<,>,\leq,\geq,=\}$). To provide semantics, we augment SiGS by a probabilistic observation function $obs_a : St\to Dist(St)$, with the idea that $obs_a(s)(t)$ gives the subjective probability with which agent a believes that the current state is t, provided that the actual current state is, in fact, s. We will often write $obs_a(t|s)$ instead of $obs_a(s)(t)$ to make this even clearer. Additionally, we lift the notation to subsets of states by defining $obs_a(T|s)=\sum_{t\in T} obs_a(t|s)$.

We require that epistemic indistinguishability, captured by $\sim_a$, is consistent with obs_a as follows: $s\sim_a t$ (when in s, the agent considers t as possible) iff $obs_a(t|s)>0$ (when in s, the agent considers t with nonzero probability). Since $\sim_a$ is an equivalence, this implies the following requirements: $obs(s|s)>0$ (due to reflexivity of $\sim_a$); if $obs(t|s)>0$ then $obs(s|t)>0$ (due to symmetry); if $obs(t|s)>0$ and $obs(s|w)>0$ then $obs(t|w)>0$ (due to transitivity).

An SiCGS extended by observation functions obs_a is called a Stochastic Observational CGS (SOCGS). An SOCGS together with a set of atomic propositions AP and a valuation function $V : AP\to 2^{St}$ is called a Stochastic Observational CGM (SOCGM). The semantics of **PATLC** extends **PATL** by the clause:$M,s\models\mathcal{K}_a^{\propto q}\varphi$ iff $obs_a(Sat(M,\varphi)|s)\propto q$. We note in passing that, for finite models, we can express classical knowledge by credences with $\mathcal{K}_a\varphi\equiv\mathcal{K}_a^{=1}\varphi$.

We propose modal operators $\mathcal{H}_a^{\propto m}\Phi$, based on the fundamental notion of *information entropy*, due to [9]. Let $X=\{x_1,\ldots,x_n\}$ be a countable set of *possible outcomes* (typically, values of a given random variable), and $Pr\in Dist(X)$ a probability distribution over X. Then, its Shannon entropy is defined as $H_S(X)=-\sum_{i=1}^{n} Pr(x_i)\log Pr(x_i)$. Clearly, Shannon entropy is minimal (and equal to 0) if p is a Dirac distribution, i.e., we are certain with probability 1 which outcome is the right one. Conversely, H_S is maximal when Pr is uniform, i.e., the subjective randomness of the system is highest. In that case, Shannon entropy coincides with Hartley uncertainty, defined as $H(X) = \log(|X|)$ [8].

Syntax. **PATLH** extends **PATL** with a family of operators $\mathcal{H}_a^{\propto m}\Phi$, where $a \in \mathbb{A}\text{gt}$, Φ is a finite nonempty subset of **PATLH** formulas, and $\propto \in \{<, \leq, >, \geq, =\}$ is a comparison operator. The reading of $\mathcal{H}_a^{\propto m}\{\varphi_1, \ldots, \varphi_n\}$ is "the uncertainty of a about the actual values of $\varphi_1, \ldots, \varphi_n$ is at least (at most, equal to, etc.) m."

Semantics. We define the semantics of **PATLH** over SOCGMs, as for **PATLK** and **PATLC**. We follow the approach of [11]. The idea is that, in order to measure agent a's uncertainty about formulas $\Phi = \{\varphi_1, \ldots, \varphi_n\}$, we take the different valuations of Φ as the "possible outcomes." Then, we take a's subjective probabilities about each possible valuation, and compute Shannon entropy for the resulting distribution.

Formally, we start by defining relation $\sim^{\varphi} \in St \times St$ that connects states with the same valuation of φ, i.e.: $s \sim^{\varphi} t$ iff $M, s \models \varphi \Leftrightarrow M, t \models \varphi$. This can be lifted to indiscernibility of states w.r.t. a set of formulas Φ in a natural way: $\sim^{\Phi} = \bigcap_{\varphi \in \Phi}^{n} \sim^{\varphi}$. Moreover, we combine syntactic and semantic indistinguishability through relation $\sim_a^{\Phi} = \sim_a \cap \sim^{\Phi}$. In other words, $s \sim_a^{\Phi} t$ iff s and t are epistemically indistinguishable and no formula in Φ can distinguish between them. Clearly, $\sim_a^{\Phi}$ is an equivalence relation.

Next, we define the relevant outcomes as the abstraction classes of $\sim_a^{\Phi}$ that are contained in the current epistemic class due to $\sim_a$, i.e.: $R_{a,s}(\Phi) = \{[t]_{\sim_a^{\Phi}} \mid t \sim_a s\}$. Then, we construct $Pr_{a,s,\Phi} \in Dist(R_{a,s}(\Phi))$ with $Pr_{a,s,\Phi}([t])$ being the (normalized) aggregate probability that a associates with the states in $[t]$ when the real state of the system is s, i.e.: $Pr_{a,s,\Phi}([t]_{\sim_a^{\Phi}}) = \sum_{t' \in [t]_{\sim_a^{\Phi}}} obs_a(t'|s)$. Finally, we can define the semantics of Shannon uncertainty modalities via the following clause: $M, s \models \mathcal{H}_a^{\propto m}\Phi$ iff $\big(-\sum_{[t] \in R_{a,s}(\Phi)} Pr_{a,s,\Phi}([t]) \log Pr_{a,s,\Phi}([t])\big) \propto m$.

4 Succinctness

The concept of *succinctness* focuses on whether there is a substantial difference in the length of encodings provided by logics L_1, L_2 for some scalable property [10, 13, 1]). In this section, we show that **PATLH** is exponentially more succinct than **PATLC**. To prove this, we follow [11] and use *formula size games (FSG)* introduced in [6]. Specifically, we demonstrate that there is a sequence of **PATLH** formulas $(\varphi_n)_{n \in \mathbb{N}}$ with length $O(n)$, such that any **PATLC** formula ψ_n with the extension as φ_n has a parse tree with at least 2^n distinct vertices, and thus that the length of ψ_n must be at least $O(2^n)$.[1]

Definition 1 (Succinctness, [1]). *Let $L_1 = (\mathcal{L}_1, \models_1)$ and $L_2 = (\mathcal{L}_2, \models_2)$ be two logical systems with sets of formulas $\mathcal{L}_1, \mathcal{L}_2$ and semantic relations $\models_1, \models_2$ interpreted over the same class of models $\mathcal{M}$. By $Sat(\phi) = \{(M, q) \mid M, q \models \phi\}$, we denote the class of pointed models that satisfy ϕ in the semantics given by $\models$. Likewise, $Sat(M, \phi) = \{q \mid M, q \models \phi\}$ is the set of states (or, equivalently, pointed models) that satisfy ϕ in a given structure M. Further, suppose $f, g : \mathbb{N} \to \mathbb{N}$ are two functions such that $f(n) = O(g(n))$ is a strictly increasing function. L_1 is exponentially more succinct than L_2 ($L_1 \not\preceq_M^{subexp} L_2$) iff for each natural number $n \in \mathbb{N}$ there are formulas $\varphi_n \in \mathcal{L}_1$ and $\psi_n \in \mathcal{L}_2$ with: (1) $|\varphi_n| = f(n)$, (2) $|\psi_n| = 2^{g(n)}$, (3) ψ_n is the shortest formula in $\mathcal{L}_2$, equivalent to φ_n on $\mathcal{M}$.*

We now adapt the one-person formula size games of [5], where the player, called the spoiler tries to synthesize a formula to discern between sets of models A and B, i.e., some $\varphi \in$ **PATLC** such that $A \models \varphi$ and $B \models \neg\varphi$.

[1] The length of formula φ is defined as the total number of symbols in φ, including atomic propositions, boolean, modal and arithmetic operators, and numbers (treated each as a single symbol).

We use Formula Size Games for proving that **PATLH** is exponentially more succinct than **PATLC**.

Theorem 1 (Succinctness of **PATLH** vs. **PATLC** (Conjecture 1 in [7])). ***PATLH** is exponentially more succinct than **PATLC**. In particular, there exists a sequence of formulas $(\varphi_n)_{n\in\mathbb{N}}$ in **PATLH** of size $O(n)$ such that for each n, any **PATLC** formula ψ_n equivalent to φ_n (i.e. true in exactly the same class of models) must have size $\Omega(2^n)$.*

Proof idea. For every natural number $n \in \mathbb{N}$, we define a formula φ_n in the language of **PATLH** and consider the shortest formula ψ_n in the language of **PATLC** that is expressively equivalent to φ_n. The existence of such ψ_n is guaranteed by properties of the Formula Size Games. Further, we construct two sets of pointed models A_n and B_n, such that $A_n \models_{\mathcal{H}} \varphi_n$ and $B_n \models_{\mathcal{H}} \neg\varphi_n$,

The idea is to use $A_n = \{M\}$ with two agents $\{a, b\}$ and the state space divided into n "layers," where obs_a is defined within each layer according to the geometric probability distribution (a.k.a. Furry distribution), and obs_b leads to states in the subsequent layer. Moreover, B_n is the set of possible variants of M obtained by removal of an arbitrary state within each layer. Then, we take: $\phi_1 \equiv \mathcal{H}_a^{\geq q}\{\mathsf{p}_1, \dots \mathsf{p}_\mathsf{k}\}$, $\phi_2 \equiv \mathcal{H}_a^{\geq q}\{\mathcal{H}_b^{=0}\mathcal{H}_a^{\geq q}\{\mathsf{p}_1, \dots \mathsf{p}_\mathsf{k}\}, \dots \mathsf{p}_\mathsf{k}\}$, and so on, for an appropriately chosen q.

We will thus have that $A_n \models \psi_n$ and $B_n \models \neg\psi_n$. Hence, in the FSG with the starting point at the vertex labelled by (A_n, B_n), the spoiler has a winning strategy given by playing the moves in accordance with the syntactic structure of ψ_n. Still, the last step remains to demonstrate that any such winning strategy requires that the length of the play of the game (starting at (A_n, B_n)) is no shorter than 2^n. □

5 Conclusion

We have established that **PATLH** (**PATL** with Shannon-entropy modalities) is exponentially more succinct than **PATLC** (**PATL** with probability-threshold modalities). In particular, as shown in Theorem 1, there is a family of **PATLH** formulas φ_n of size $O(n)$ for which any equivalent **PATLC** formula must have size $2^{\Omega(n)}$. Our proof uses a one-person formula-size-game technique adapted to the **PATLH/PATLC** setting, filling a gap in the literature on logical succinctness, building on and confirming conjectures from the previous study of **PATLH/PATLC**. This result demonstrates that entropy-based specifications can be dramatically shorter than purely probabilistic ones. This means using Shannon-entropy operators yields much more compact specifications for probabilistic knowledge goals than traditional probability-threshold operators. Additionally, by adapting the one-person formula-size-game framework to the probabilistic temporal setting, we are providing a novel proof technique for succinctness. Last, but not least, however, due to expressiveness constraints demonstrated in [7], while **PATLH** allows concise encodings of goals, designers must account for the potential cost in verification complexity when choosing between logics.

Potential future research directions include investigating succinctness with other uncertainty measures (such as Renyi or min-entropy) and their impact on specification size in temporal logics; examining exponential succinctness phenomena in richer logics, for example combining entropy operators with epistemic (knowledge) modalities or common knowledge; and finally, developing prototype tools or case studies to gauge how much the succinctness of **PATLH** helps in practice, and to find heuristics for handling entropy-based specifications.

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