Procedural Semantics and Hyperintensionality in Transparent Intensional Logic

1. Introduction

The logical analysis of natural language requires a formal framework capable of distinguishing meanings at a level finer than truth-conditions. Extensional and intensional semantic theories fail to capture such distinctions, motivating the development of hyperintensional semantics.

Procedural Semantics are fine-grained semantics. For example, logically equivalent propositions are distinct between each other. Marie Duži, Bjorn Jespersen and Pavel Materna, in their book *Procedural Semantics for Hyperintensional Logic* (2010), develop a general context of procedural semantics through Transparent Intensional Logic, which is a hyperintensional system formulated in a typed λ-calculus, designed for the logical analysis of natural language (Kovář et al., 2010, p. 69).

The aim of this paper is to present Procedural Semantics within TIL, focusing on constructions as procedures, hyperintensional individuation, and propositions as structured procedures. A brief suggestion is made, at the end of the paper, regarding the possibility of extending the framework toward incorporating distinctions in illocutionary force.

2. Transparent Intensional Logic

Transparent Intensional Logic (TIL) is a system of formal logic, originally developed by Pavel Tichý. It is a typed λ-calculus with a ramified type hierarchy, extending Church's system to include intensional and temporal parameters (Vokorokos et al., 2017, p. 420). Tichý in his book *The Foundations of Frege's Logic* (1988, p. 1-15) introduces a third level in logical analysis, along with extensions and intensions which are already been used in other formal systems, which are called constructions. The meaning of an expression is now seen not a static value, but as the construction (Holster, 2003, p. 7).

The program rests on three pillars: Referential transparency, compositionality, and hyperintensionality. Referential transparency rejects "reference shift," ensuring that expressions retain their meaning in all contexts. Compositionality guarantees that the meaning of a complex expression is fully determined by its parts. The analysis of natural languages requires more complex processing than an extensional framework, or even an intensional one can provide. A hyperintensional context, within which distinction between not only co-extensional, but also logically equivalent or co-intensional terms can be made, is one of high granularity. For example, the expressions "Every student passed" and "No student failed" are logically equivalent, but in a hyperintensional context, they have distinct semantics.

This becomes even more evident in propositional attitudes, e.g.: Believe (a, A) does not imply Believe (a, B), even when A ≡ B (Duží et al., 2010, p. 422).

TIL employs a typed λ-calculus with partial functions, extended into a ramified type theory. It introduces four primitive constructions:, Variable, Trivialization, Composition and Closure, as the building blocks of meanings. The Trivialization of *X*, denoted '^{0}X', constructs *X* without the mediation of any other constructions. Composition is the procedure of applying a function *f* to an argument A to obtain the value (if any) of *f* at A. Closure is the procedure of constructing a function by abstracting over variables; i.e., the procedure of abstracting, or extracting, a function from a context, as when abstracting $\lambda x(\phi\ x)$ from $\phi\ (a)$.

These permit precise analysis of intensional and hyperintensional contexts, including de dicto/de re distinctions, attitude reports, and semantic puzzles that resist model-theoretic approaches.

The objectual base *B* consists of the following atomic types:

ο the set of truth-values {**T, F**};

ι the set of individuals (the universe of discourse);

τ the set of real numbers;

ω the set of logically possible worlds (the logical space)

Duži, Jespersen and Materna develop a general context of procedural semantics through TIL. They conceive constructions as abstract, objective procedures, of one or more steps. Just as an arithmetic calculation takes numbers, processes them and yields other numbers, so constructions are, semantically speaking, calculations whose results may be, for instance, truth-values, truth-conditions, numbers, properties, as well as other calculations (Duží et al., 2010, p. 42-43).

3. Constructions as Procedures

For Pavel Tichý, the fundamental underlying concept of the entire TIL program is the function, but not in the sense of contemporary mathematics as a mapping; rather, it is understood in terms of construction.

In the modern conception of functions, the distinction between functions is extensional. Two expressions may correspond to different constructions but determine the same function, for example: the expressions $2x^2 + (5 - x^2)$ and $x \cdot x + 5$ yield the same values (so they are considered identical) but differ procedurally. However, this does not hold for constructions. This distinction underlies the hyperintensional character of TIL. Extending this perspective to natural languages, for Duží et al. (2010, pp. 18–19), a construction understood in terms of process constitutes the extra-linguistic entity that mediates between an expression and the object it denotes, corresponding to Frege's notion of *Sinn*.

It is further clarified that what mediates between the expression and that which it expresses (the denotation) is a construction. The construction is an abstract, objective process that exists independently, forming the underlying foundation, and is subsequently encoded in natural languages. However, two logically or intensionally equivalent expressions do not necessarily share the same construction.

4. Procedural Semantics within TIL

As B. Jespersen (2019) argues, the proposition is considered as structured and is identical to the procedure itself, e.g. that of predication. The procedure is conceived within the context of TIL as an abstract, higher-order structure of high granularity. It is something displayed, not executed, because execution would mean that the proposition is the result of the process, which is not the case, since it is the procedure itself. The parts belong to specific λ-terms, and different types of λ-terms express different kinds of logical procedures.

We can see an example of the symbolization of an analytical proposition within the context of procedural semantics through TIL: "2 is even" is symbolized as $[{}^{0}\text{Even}\ {}^{0}2]$.

The atomic, or one-step, procedure ${}^{0}\text{Even}$ produces the characteristic function of the set of even natural numbers. The atomic procedure ${}^{0}2$ produces the number 2. If the types match with each other, the procedure is displayed and executed, like above. If not, it is displayed but not executed (e.g. 'Blue is an even number'). This, seen as a procedure within the context of TIL, can be displayed and although it cannot be executed, so to take a truth value, it can still serve as the meaning of the sentence.

For the analysis of empirical propositions, two empirical λ-terms (variables) are introduced, possible worlds and time. The proposition is a function of the form **P**: w → (t → o), where **w**: the class of possible worlds, **t**: the class of times, **o**: the set {T, F}. This means that the truth value of a proposition results from the application of the world(s) and time(s) that it is or is not the case. First, we apply the proposition to a world and then to a point in time in that world, in order to find its truth value. We can see an example of the symbolization of such a proposition:

"Plato teaches" is symbolized as: $[\lambda w\, [\lambda t\, [[[{}^{0}\text{Teach}\ w]\ t]\ {}^{0}\text{Plato}]]]$

or more simplified: $[\lambda w \lambda t\, [{}^{0}\text{Teach}_{wt}\ {}^{0}\text{Plato}]]$

i. Application of 'teach' (w → (t → (a → o))) to w, in order to wander through the various worlds

ii. Apply the timing of step (i) to t, so that we wander through time and have a set of atoms (α → o).

iii. Apply the whole step (ii) to the individual Plato, so that we have a truth value.

However, the proposition itself is the procedure, not its result. Procedural semantics allows for distinctions between expressions that share the same truth-conditions. For example:

"The temperature is above zero" and "It is not freezing"

These expressions may correspond to different constructions, so they belong to different types of λ-terms. Thus, an agent may grasp one construction but not another logically equivalent one, avoiding logical omniscience.

5. TIL and Illocutionary Force

According to Frege's distinction between a sentence's force and its content, expressing a propositional content is one thing, and doing so with a specific kind of force – assertive, interrogative, imperative, etc. – is quite another. Recently, there has been renewed discussion challenging the issue known as the *"Frege–Geach Point",* which demands that the same sentence can be both affirmed and not affirmed at the same time, for example when it occurs as a premise in a hypothetical argument (Geach, 1965, p. 449). Hanks (2007), Recanati (2019), and Bronzo (2019) express objections to this. They argue that sentences incorporate some kind of illocutionary force, so that even though they share the same content, they count as different sentences if they have different force.

TIL maintains a strict distinction between propositional content and illocutionary force, treating the latter as external to semantics (Duží et al., 2010, p. 15). "The syntactic difference between propositions does not reflect any difference in the logic of the two linguistic propositions" (Tichy, 1978, pp. 278, 275). Thus: 'Plato teaches' (1) and 'Does Plato teach?' (2)
share the same propositional content, namely that Plato teaches, but in (1) this is conveyed with assertive force, whereas in (2) it is conveyed with interrogative force. It is symbolized on both occasions as such:

$$[\lambda w \lambda t\ [{}^0Teach_{wt}\ {}^0Plato]]$$

However, given the fine-grained structure of constructions, we raise a research question: whether force distinctions could also be represented procedurally. Such an approach would extend the framework while preserving its procedural architecture.

8. Conclusion

In sum, TIL proposes a unified, fine-grained semantics grounded in constructions, offering a principled alternative to model-theoretic frameworks and advancing the project of logical analysis of natural language. It provides a hyperintensional framework grounded in a typed λ-calculus, where meanings are identified with structured procedures.

This approach captures distinctions of high granularity, distinguishes logically equivalent expressions, and provides a principled account of propositional structure. At the same time, it opens the possibility for further extensions, including the incorporation of illocutionary force.

Finally, it is worth noting that TIL has also found applications in computer science and artificial intelligence, particularly in areas such as natural language processing and intelligent question-answering systems, where its procedural and hyperintensional structure supports fine-grained semantic analysis (Duží & Fait, 2021).

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