Aristotle versus Euclid: Syllogising the *Elements* and the epistemological status of mathematics

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Abstract

From the 17th century, an ideal system of knowledge had to be built on clear axioms and developed *more geometrico*, modelled on Euclid's *Elements*. Yet only decades earlier, the *Elements* themselves were debated regarding their ability to provide unquestionable knowledge. In the 16th century, Aristotelian doctrine with syllogistic logic remained the model of science. As doubts arose about mathematical certainty, attempts were made to formalise the *Elements*, reducing them to Aristotelian syllogisms. This paper presents such attempts together with a syllogistic formalisation of Proposition I.5 and argues for the crucial importance of "syllogising Euclid" for history of mathematics and logic alike.

1 Introduction

After what is now often called the "epistemological shift", in the era of Descartes and Spinoza, what was considered knowledge had to be clear and distinct – an ideal system of knowledge, to stand firm, had to be built on clear axioms and then be developed *more geometrico*, that is, be modelled on the canonical example of Euclid's *Elements*.

A less familiar part of the story is that only a few decades earlier, the *Elements* themselves were a subject of a philosophical debate concerning their ability to provide certain and unquestionable knowledge. In the sixteenth century, despite the rising significance of mathematics and natural sciences, what was considered to be the model of science as such was still the Aristotelian doctrine, with the syllogistic logic as its organon. Hence, when during the so-called *Quaestio de certitudine mathematicarum*[1] [*Question on the certainty of mathematics*] doubts were raised about the epistemological status of mathematics, attempts were made to formalise Euclid's *Elements* by rendering the proofs of Euclidean propositions using the formal syllogisms of Aristotle.

These attempts, relatively frequent from the second half of the sixteenth century to the first half of the seventeenth century, were thereafter abandoned and quickly forgotten – the reason for which remains unstudied. Below, I review several key attempts at "syllogising Euclid". I start by mentioning the historical roots of this movement – most notably Avicenna and Algazel – to then introduce the early modern attempts. After this, I present an example of a syllogistic formalisation taken from the *Analyseis geometricae* of Herlinus and Dasypodius and provide a brief reconstruction of it. Thereafter, I discuss the philosophical underpinnings of the "syllogisation programme": I argue that the nature of the modern attempts was fundamentally different from the earlier ones, and that this difference was precisely due to the rapidly changing epistemological status of mathematics in the sixteenth and seventeenth centuries. Linking syllogising the *Elements* to debates about the certainty of mathematics will allow me to consider questions such as why the syllogisation movement has received such extensive interests from philosophers but only for a very limited period of time, and also how and why the movement

[1] For an extensive overview of this debate see [10].

was crucial for the later development of mathematics, logic, and formal sciences in general. I end by proposing possible directions for future research.

2 Attempts at formalising Euclid with syllogistic

The earliest known remark on a syllogistic treatment of Euclid is Avicenna's formalisation of Euclid I.1. present in his *Treatise on logic* [1, p. 36]. This formalisation of Avicenna was reproduced later by Algazel and was available in the Latin West under the title *Logica et philosophia Algazelis* [9]. However, despite its availability in the Middle Ages, no attempts of medieval scholars to syllogise Euclid have been identified so far. The first known European attempt appears only in the early modern period, in the first half of the sixteenth century, and is that of Alessandro Piccolomini's formalisation of Euclid I.1 [12].[2]

Quite a few attempts at syllogising Euclid were made later, although all were made before the end of seventeenth century. Unfortunately, virtually all of them remain untouched by modern scholarship and – being in Latin and Ancient Greek – inaccessible to most of the modern scholars due to lack of translations. The ones I have identified are:

1. *Analyseis geometricae sex Librorum Euclidis* (1566, [8]) by Christianus Herlinus and Conrad Dasypodius

2. *Euclidis Elementorum Libri XV* (1574, [5]), an edition of *Elements* by Christopher Clavius

3. *Cursus Mathematicus* (1634, [7]) by Pierre Hérigone

4. *Analysis Aristotelica ex Euclide restituta* (1658, [16]) by Erhard Weigel

5. *Mathematicae Lectiones* (1683, [2]) by Isaac Barrow[3]

Most of these works replicate what was present in Avicenna and Piccolomini, offering only a formalisation of Euclid I.1. On this basis, however, their authors argue that *all other propositions can be formalised similarly*. Such are the works of Clavius, Hérigone, and Barrow.[4] The work of Weigel goes a little beyond this, mentioning the formalisations of I.1 and offering a formalisation of I.32. The most comprehensive work are the *Analyseis* of Herlinus and Dasypodius, which offer formalisations of all the propositions from the first six books of Euclid.

In *Analyseis*, the authors follow the schema proposed by Proclus [13, p. 159], according to which every complete Euclidean proof can be divided into its constituent parts. I follow the same schema here. Below, I cite the proposition, exposition, definition, and construction parts of Proposition I.5 as it appears in Euclid. These parts have the same content both in Euclid and in the *Analyseis*.[5] Subsequently, I give the first step of the demonstration part as it appears in Euclid along its formalisation done by Herlinus.

[2] For an analysis of Piccolomini's attempt see [11].

[3] An interesting attempt to syllogise not precisely Euclid, but a calculation of mathematical proportion, is present in the journal of Isaac Beckmann [3, p. 170–172]. There is also a work *Universalia Euclidea* by Johannes Chrisophorus Sturmius [15], which contains a part on Euclid and a separate part on syllogisms. Sturmius does not venture into syllogising Euclid, but states that mathematical and logical demonstrations have ultimately "the same metaphysical source".

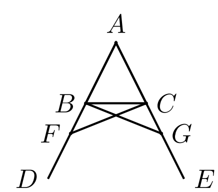
[4] An analysis of I.1., as formalised in Clavius, Hérigone, and *Analyseis* of Herlinus and Dasypodius is given in [4].

[5] I cite an English translation of Euclid by T.L. Heath [6]. The translation of Herlinus' text is mine.

3 Example syllogisation

Text of Euclid

> *Proposition.* In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.
>
> *Exposition.* Let ABC be an isosceles triangle having the side AB equal to the side AC; and let the straight lines BD, CE be produced further in a straight line with AB, AC.
>
> *Definition.* I say that the angle ABC is equal to the angle ACB, and the angle CBD to the angle BCE.
>
> *Construction.* Let a point F be taken at random on BD; from AE the greater let AG be cut off equal to AF the less; and let the straight lines FC, GB be joined.
>
> *Demonstration, step I*: In fact, since AF is equal to AG, and AB to AC, the two (straight-lines) FA, AC are equal to the two (straight-lines) GA, AB, respectively. They also encompass a common angle FAG. Thus, the base FC is equal to the base GB, and the triangle AFC will be equal to the triangle AGB, and the remaining angles subtended by the equal sides will be equal to the corresponding remaining angles. (That is) ACF to ABG, and AFC to AGB.



F. 1 1: Triangles

Syllogism for step I

> Any two triangles which have two sides equal to each other's two sides, and the angle contained by the equal straight lines equal to each other, will also have the base equal to each other, as well as the remaining angles, which are subtended by the equal sides.
>
> Triangles BAG and CAF have two sides, BA and AG, equal to two sides, CA and AF, each to each; side BA to side CA and side AG to side AF. They also have a common angle BAG. Therefore: triangles BAG and CAF have the base BG equal to the base CF, the angle ABG equal to the angle ACF, and the angle AGB equal to the angle AFC.
>
> Explanation: The major premise is the Fourth Proposition. The first part of the minor premise is the hypothesis. The second is known from the construction. The third is known by itself.

This first syllogism is the most elaborate in a series of four. The structure, repeated in all the others, is as follows: *the major premise* is an abstract statement consisting of two parts – one attempting to define the subject of the sentence, and another attempting to define what is predicated of this subject; *the minor premise* states that the case in question meets the requirements of being the subject of the major premise; *the conclusion* is that therefore, the predicate of the major premise can be predicated of the subject of the minor. After that, there follows an explanation which clarifies the sources of our knowledge about the truth of the premises.

Following the Aristotelian guidelines (see [14]), this syllogism can be interpreted as follows. We define a pair of triangles M as having certain properties and then say that these properties necessarily lead to other properties P. Then, we admit also that the triangles in question, let us call them S, have the basic properties of triangles M. Then, we conclude that they also have the properties P. The structure is therefore that of a syllogism *Barbara*: *Major premise:* Every M is P; *Minor premise:* Every S is M; *Conclusion:* Every S is P. In fact, this structure is replicated in *all* syllogistic formalisation in *Analyseis* and other works as well.

4 The epistemological status of mathematics and logic

Two fundamental features of the "syllogising movement" are to be observed. First, it appeared surprisingly late, only in the sixteenth century, although examples of such syllogistic formalisations were known before due to the work of Avicenna and Algazel. Second, it quickly evaporated from the intellectual map of Europe, leaving virtually no trace of direct influence on later thinkers.

To elucidate why that is the case, I propose to consider the rapid advance of mathematical knowledge in the sixteenth century, the resulting shift in understanding of what mathematics is about, and an urgent need to confront this new understanding with the Aristotelian doctrine, whose significance was diminishing but which was still guiding the understanding of what a science is. In this context, a clash between Aristotelian formalism (syllogistic) and mathematical formalism (Euclid) became inevitable, with its natural battleground being syllogisation of mathematics; and it is this context that explains the extensive interest given by scholars to this issue over a relatively short period of time.

To support this argument, one can observe that with modern attempts, the understanding of what is the sole nature of syllogisation changed. Avicenna, confident in the Aristotelian concept of science, was not stating that the propositions of the *Elements* can merely be *rewritten* by syllogisms, but that they *are* syllogisms, only implicitly stated. From Piccolomini onwards, by contrast, the initial assumption was that the propositions are *not* in themselves syllogisms, and they need to be rewritten in order to obtain a syllogistic form. This change in understanding meant that syllogising the *Elements*, for a while, became crucial for establishing what is the relation between logic and mathematics. From the syllogistic form of Euclidean propositions – or lack thereof – authors would infer that Euclid is compatible or incompatible with Aristotle.

The issue, however, does not end here: while some authors taking part in the *Quaestio* dispute were using syllogistic formalisations to prove that Aristotelian logic *can* accommodate the logic of mathematics and hence the mathematical reasoning satisfies the Aristotelian definition of science, others were arguing the opposite: that Aristotelian doctrine *cannot* be satisfactorily used to render Euclid's Propositions. But, since the significance of mathematics was rising while that of Aristotelian logic was diminishing, their final conclusion was not that mathematics cannot be said to be a legitimate science, but rather that *it is the Aristotelian definition of what constitutes a legitimate science that is to be dispensed with.*

Syllogising the *Elements* proves to be of importance for both the history of philosophy and the history of mathematics for two reasons: first, it played an important role in undermining and then dispensing with the Aristotelian doctrine; second, the effect of these discussions, being the liberation of mathematics from the chains of the Aristotelian doctrine, was both a crucial and a natural step required for mathematics to continue its development in the decades to come. A closer study of the surveyed attempts suggests at least three areas of further investigation:

1. While the rise of the syllogisation programme was linked to the *Quaestio* debate, the decline of it, and its subsequent oblivion, by contrast, seem to be due largely to extra-scientific factors such as the works that syllogise Euclid being excluded from school curricula.
2. A close but hitherto unstudied connection between syllogistic formalisations and the early

attempts at constructing symbolic notation seems to exist – especially the text of Weigel [16] suggests that it was the cumbersomeness of syllogistic formalisations that prompted mathematicians and philosophers to develop more straightforward formalisms that eventually superseded the Aristotelian one.
3. From the contemporary perspective, it seems that the project of formalising Euclidean geometry in syllogistic faces not only such practical difficulties but also more essential problems related to the fact that the language of Aristotelian logic is not well suited to capturing logical forms that are central in mathematics, such as relational structures or more complex quantificational patterns. One may ask whether the limitations in the expressive powers of syllogistic played a role in abandoning the syllogisation programme; conversely, one may ask whether the syllogisiation programme was not crucial in denouncing the expressive limitations of syllogistic.

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