

Exploring Aristotelian Syllogistic in First-Order Logic: An Overview of the History and Reality of Ontological Commitments

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The purpose of the paper is to answer the question of what additional existential premises are needed in order to render Aristotelian syllogisms provable in First-Order Logic and to give an overview of how the issue of ontological commitments in syllogistic was handled throughout the history. In contemporary discussions concerning the history of logic, there is a widespread assumption that the Aristotelian syllogistic, as it is the case with the modern formal logic, did not allow for the use of empty terms. The issue is however much more complex, and ontological commitments of syllogistic were discussed extensively throughout the history. In the paper, we relate these discussions, covering four main areas: the logic of Aristotle, Arabic logic, medieval European logic, and later discussions up till the emergence of modern formal logic, as well as provide our own view of the issue.

The question of empty terms and existential import in syllogistic became first apparent when Jan Łukasiewicz [12] claimed that Aristotle supposes all terms used in syllogisms to be non-empty. This view seems to be an orthodox way of interpreting up till now and was held by many scholars [14, p. 7], [22, p. 144], [24, p. 343-4]. Kneales in their *Development of Logic* state that “In order to justify Aristotle’s doctrine as a whole it is necessary, then, to presuppose that he assumed application of *all* [original emphasis] the general terms with which he dealt.” [11, p. 60]. The case is also true for Patzig, who writes that “The expression ‘one must examine the set of subject (predicate, contrary) terms of $S(P)$ ’ clearly presupposes that in each case these sets have at least one member.” [19, p. 6].

However, this viewpoint was never held by Aristotle himself, nor it was explicitly addressed in any of his works, let alone those concerned with syllogistic. Malink [13, p. 82] says that “the question of whether or not an individual falls under a term seems to be irrelevant in *Prior Analytics* 1.1-22.” In fact, a view that allows for the emptiness of terms when interpreting Aristotle is getting more and more advocates, and the discussion is ongoing [18]. The nonemptiness assumption, in turn, is said to be forced by attempts to render Aristotle using the modern notation [16, p. 74]. Scholars that opt for this view mostly refer to fragments from *Prior* and *Posterior Analytics* where Aristotle is speaking about a “goat-stag” as a syllogistic term [3, p. 243], [13, 81]. For example: “(...) you may know what the account or the name signifies when I say goat-stag, but it is impossible to know what a goat-stag is (...)” (*Posterior Analytics* 92b6-8). As a goat-stag is a nonexistent, from its presence it is then argued that Aristotle must have been aware of such a possibility and thus his theory have to account for empty terms as well. In this context, Wedin [25, p. 179] is also quoting *Categories* 13b12-36, where Aristotle states that both “Socrates is sick” and “Socrates is healthy” are false in case Socrates does not exist, but “Socrates is sick” and “Socrates is not sick” become opposites in that case. From this it is argued that Aristotle is claiming the existence of a subject as a truth condition and thus must be aware that additional existential premises are required for some statements to be true.

The comments above cannot be easily generalized to syllogistic, but they point out the fact

that Aristotle was in fact aware of the possibility of empty terms being taken into consideration. The question of how Aristotle originally intended his syllogistic to treat empty terms remains open. Here, it will suffice to say that he does not make any explicit statements about it and (non)emptiness does not yet emerge as an issue. Nevertheless, when it comes to modern discussions, we can observe a tendency leading from the one-sidedness of first interpretations to a more nuanced view.

Whatever might be said about Aristotle, empty terms were widely discussed both in Arabic and in medieval European logic. The first one acknowledged to explicitly talk about the existential import is Al-Farabi [8, p. 39], although in his Syllogism, he does not talk about empty terms at all, and the discussion is confined to categorical statements. Avicenna continues to explicitly talk about existential assumptions [17, p. 142]. He also does some explicit remarks on syllogistic and require that negative propositions in syllogisms have an existential import as well [3, p. 293]. Moreover, the existence he talks about is not restricted to real existence as in Al-Farabi, but apart from existence *in re*, existence *in intellectu* is also considered [7, p. 90], and this line of thought continues also in works of Averroes [4, p. 361].

In Europe, historically speaking, the question of existential import was not addressed explicitly until the rise of nominalism, with William of Ockham being the first one to pronounce it [6, p. 420]. In general, the discussion of empty terms was virtually nonexistent before the nineteenth century [18]. Early Scholastics, such as Peter of Spain, have never considered it neither with respect to categorical statements nor to syllogistic [6, p. 417].

Ockham maintains that the truth conditions of both affirmation and denial are disjunctive, with an existence of a subject and a predication for the first, and a lack of those for the latter [1, p. 392-3]. He does not consider existence *in intellectu* and requires every subject of an affirmative statement to exist *in re* [3, p. 302]. The thing worth mentioning about him is that his treatment of existential assumptions tends to be conditional at times – when talking about the *dictum de omni et nullo*, he states that for an example affirmative sentence to be true, its subject need not always exist, but it suffices only that the sentence is true whenever it does exist [23, p. 42]. Thus, he can be viewed as a precursor of the modern notion of the universe of interpretation, with objects existing externally being the only possible interpretation. Buridan alike requires the subject of every true affirmative sentence to exist *in re* and makes this claim more explicit [9, p. 26]. He is, however, sceptical about Ockham’s conditional approach, and the existence of a subject is to be read verbatim [23, p. 42]. From both points of view, talking about nonactual beings existing somehow is forbidden and being regarded as a nonsense [10, p. 159].

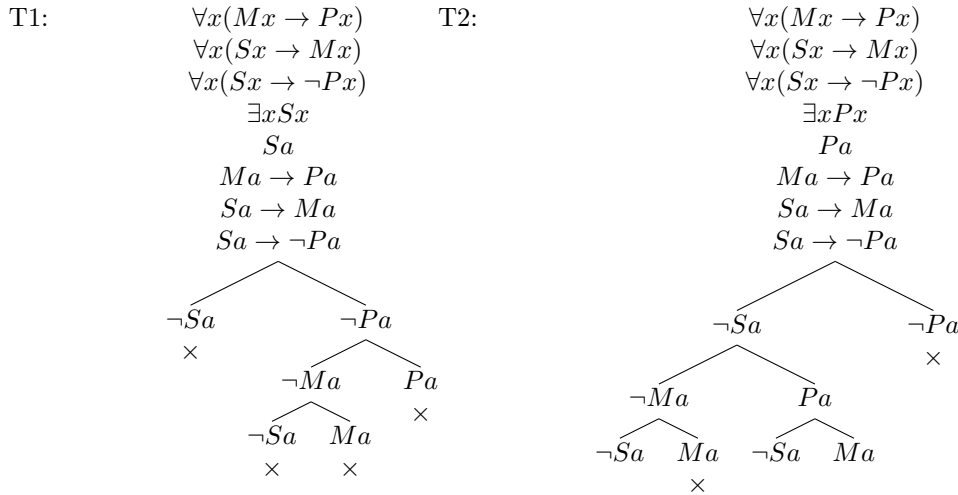
Thus, even if we agree on the tacit existential assumption in Aristotle, this view was certainly not held by the medieval logicians above, by which empty terms were discussed fervently [10, p. 143]. The *Ars Logica* by John of St. Thomas can be regarded as the culmination of this trend. John is holding the doctrine of existential import developed by nominalists, and his theory is greatly resembling the modern notion of the universe of interpretation, with things *in intellectu* considered as well as those existing in the past or in the future [6, p. 420]. And while he was talking only about categorical sentences, Leibniz was the first one to allow for terms to be systematically interpreted as things *in intellectu* in his syllogistic [15, p. 292].

Nevertheless, serious and detailed logical investigations of existential import were altogether abandoned in the third decade of the sixteenth century, and the discussions ceased [2, p. 147]. Some authors point out that the invention of Venn diagrams in 1881 helped to make the issue more explicit again [26, p. 416], which would be at least intuitively true with respect to syllogistic as well. Certainly, the development of Boolean algebra sparked a renewed interest, with such authorities as Peirce and Russell speaking up [26, p. 416], although they comment

only on existential import in general, without making reference to syllogistic. Boole himself refrains from making any direct comparison between his system and the one of Aristotle [5, p. 226]. The first one acknowledged to state that Aristotle's syllogistic requires its terms to be non-empty was Śleszyński [21], and the widespread popularity of this view stems from the works of Łukasiewicz [20, p. 1-2].

Thus, the historical development of the issue of empty terms is twofold. First, we can observe a rising awareness of the empty terms as an issue that needs to be covered – irrelevant in Aristotle, present in the Middle Ages, and substantial in the modern interpretations of Aristotle's work. The difference is that up till Łukasiewicz it was discussed either with respect to the validity of the Logical Square, as in the medieval and early modern period, or with respect to categorical statements in general, as when the Boolean algebra emerged, and only with the works of Łukasiewicz the discussion turned to syllogistic as such. Secondly, in parallel with the above, the development of the notion of the universe of interpretation can be traced, beginning with the works of Ockham and getting more and more pronounced, with Leibniz being the first one to allow for the *in intellectu* interpretation, and Boolean algebra stating the idea explicitly.

Building upon this rich historical background we may now turn to our main research question, which is this: If we consider a proof of a given syllogism, are there any additional existential premises required, besides the premises of the syllogisms itself, in order to prove it in First-Order Logic?



There are different answers to this question possible, which allow dividing valid Aristotelian syllogisms into three groups. To exemplify their members, we use syllogisms of the first figure:

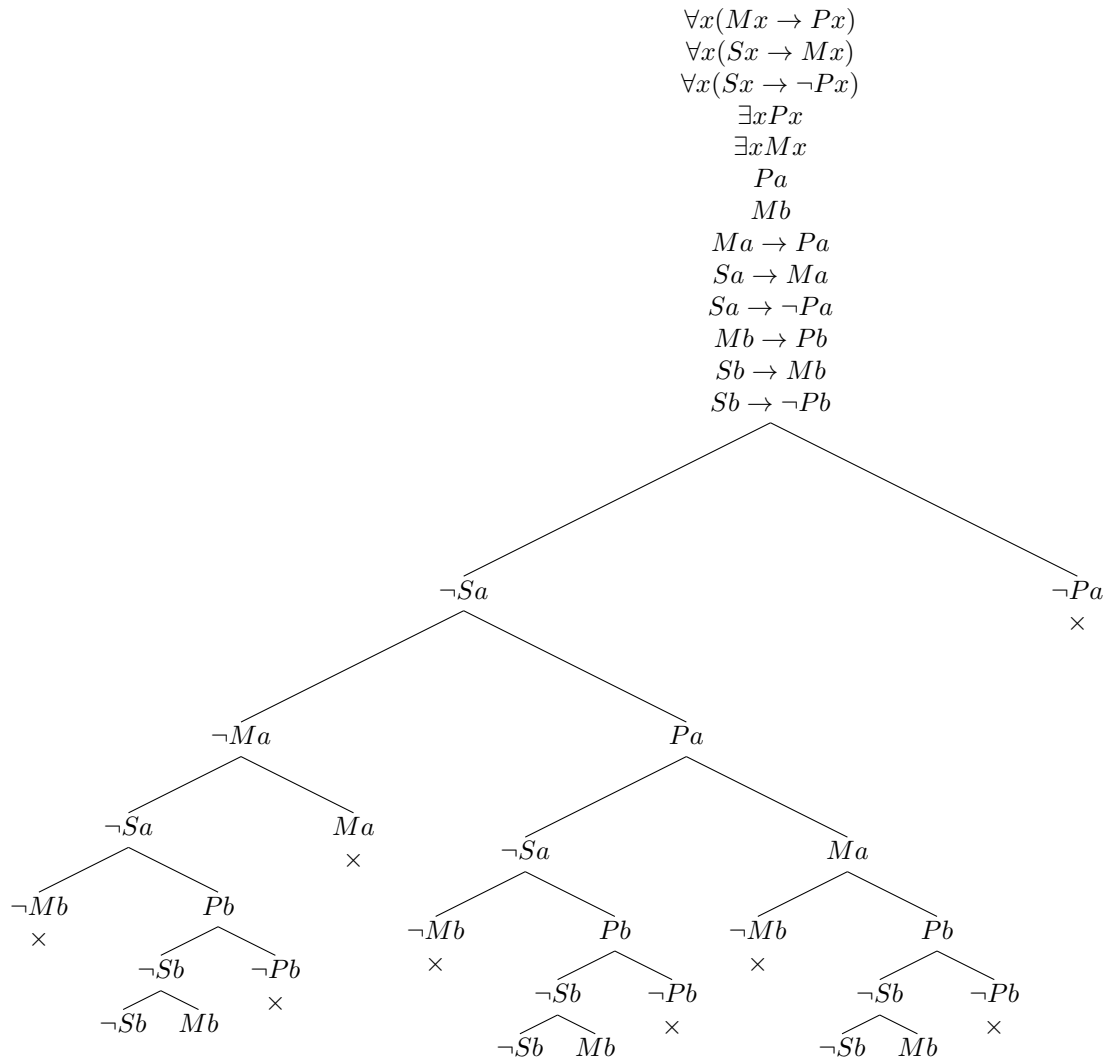
- G1 No additional premises are needed, as no special ontological commitments are required; this is the case for syllogisms in which both premises and the conclusion are general sentences (*Barbara*, *Celarent*).
- G2 No additional premises are needed, as required ontological commitments are addressed by a particular premise (*Darii*, *Ferio*).
- G3 Additional existential premise is needed, as required ontological commitments are not warranted by the premises of the syllogisms (*Barbari*, *Celaront*).

Furthermore, for the syllogisms in the third group, only one extra existential premise is necessary to prove them in First-Order Logic. Specifically, to demonstrate these syllogisms, we only need to assume the non-emptiness of one of the three terms that make up the syllogism.

Let us consider a member of the G3 group, the syllogism *Barbari* (for simplicity, we use analytic tableaux as the proof method). T1 above is an analytic tableau for *Barbari*, employing one additional existential premise, $\exists xSx$. It is easily seen that without it, the tableau will not close, while with it the tableau does close, thus forming a proof of the syllogism in question. If we add to the *Barbari*'s original premises any other existential premise, the tableau will not close; T2 is an example involving the premise $\exists xPx$.

Moreover, only addition of the $\exists xSx$ premise allows to prove *Barbari* in FOL. Consider T3, the unsuccessful attempt, with two existential premises added ($\exists xPx, \exists xMx$):

T3:



The same holds for all the other syllogisms in the G3 group. However, it is not always the minor terms that needs to be non-empty in order to prove a G3 syllogism: this depends on the figure.

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