Quid Verificabit Ipsos Verificatores?
A Model for Self-Applicable Exact Truthmaking

Simone Picenni

University of Bristol,
ERC Starting Grant "Truth and Semantics" (TRUST 803684)

July 9, 2022
PLS 13, Volos (Greece)

Structure of the presentation

- Exact truthmakers: states and relevance
- Exact truthmaking semantics
  - Exact truthmakers
  - State spaces
  - Exact truthmaking models
- Exact-truthmaking and self-applicability
  - Truth in hyperintensional contexts
  - A semantics "turning to itself"
  - Truthmaking paradoxes
- A model for self-applicable exact truthmaking...
- ... and some important facts about it

Informal notes:

Exact truthmaker (resp, falsemaker) of φ: a **state** that makes φ true (resp, false) while being **wholly relevant** to its truth (resp, falsity).

State. . . : *at least* something that decides a (possibly proper) subset of the sentences of a language. So, anything that does the job:

- State description-like (Carnap [5], van Fraassen [26], Leitgeb [14])
- Tarskian structure-like (Niiluoto [20], Tennant [25])
- State of information (Kripke [13])
- Situation (Barwise and Perry [3], Kratzer [11])
- State of affairs, fact (Wittgenstein [27], Russell [22], van Fraassen [26])

we do not require states to be consistent, and we consider these *things* abstractly.

Crucial things:

- States are truthmakers [15, 6].
- States can be endowed with a *parthood* relation – i.e.: states can be divided into parts, and be parts of states [27, 11, 12].

Wholly relevant...

- There may be parts of truthmakers that contribute to the determination of the semantic value of a sentence, and others that do not.
- *Exact* truthmakers are such that they don't have parts that do not contribute to the determination of the semantic value of a sentence they make true.

Situation: it rains and it is cold.

- Wholly relevant to the truth of "It rains and it is cold"
- "It rains": true, but not *exactly* true at it – part of it is irrelevant (the temperature).
- Wholly relevant to the truth of "It rains, or it rains and it is cold" (vs *minimality*)

Exact Truthmaking Semantics

The concept of *exact truthmaking* gives rise to a fine-grained semantics, *exact truthmaking semantics*, in which we individuate semantic content of sentences by means of their exact truthmakers.

State space

Recall crucial aspects of *statehood*

- States are truthmakers [15, 6].
- States can be endowed with a parthood relation [11].

Definition (State space)

A **state space** is an ordered pair $(S, \sqsubseteq)$ such that

- S is a non-empty set (of objects that can act as truthmakers)
- $\sqsubseteq \subseteq S \times S$ is a partial order (i.e.: a reflexive, antisymmetric and transitive relation) on S

We don't require $(S, \sqsubseteq)$ to be a complete lattice.

Exact truthmaking models

Definition

An **exact truthmaking model** for a classical first order language $\mathscr{L}$ is an ordered tuple $(S, \sqsubseteq, A, |\cdot|; D)$ such that

- $(S, \sqsubseteq)$ is a state space.
- $A \subseteq S$, the set of "actual" states, is a *downward closed* non-empty subset of S – that is: if $s \in A$ and $t \sqsubseteq s$, then $t \in A$.
- $|\cdot| \subseteq \mathscr{L} \times (\wp(S) \times \wp(S))$ is a function sending atomic sentences of the language $\mathscr{L}$ into ordered pairs of sets of states.
- $D \neq \varnothing$ is a set (the set over which our quantifiers will range).

Assumptions:

- In order to express the truth conditions of quantified sentence, we must provide a collection of objects on which the variables bound by the quantifiers of our language vary, a domain of discourse [see 9].
- for reasons of simplicity, from now on we will use a language *à la* Robinson, in which each object of the domain has a constant that denotes it [see 4].

Recursive Clauses

Definition

1. $s \Vdash \mathsf{P}(\mathsf{t}_1, \ldots, \mathsf{t}_n)$ iff $s \in proj_1|\mathsf{P}(\mathsf{t}_1, \ldots, \mathsf{t}_n)|$, and
   $s \dashv\mid \mathsf{P}(\mathsf{t}_1, \ldots, \mathsf{t}_n)$ iff $s \in proj_2|\mathsf{P}(\mathsf{t}_1, \ldots, \mathsf{t}_n)|$
2. $s \Vdash \neg\varphi$ iff $s \dashv\mid \varphi$, and $s \dashv\mid \neg\varphi$ iff $s \Vdash \varphi$
3. $s \Vdash \varphi \wedge \psi$ iff there are $s_1, s_2 \in S$ such that
   $s_1 \Vdash \varphi \;\&\; s_2 \Vdash \psi \;\&\; s_1 \sqcup s_2$ is defined, and $s = s_1 \sqcup s_2$, and
   $s \dashv\mid \varphi \wedge \psi$ iff $s \dashv\mid \varphi$ or $s \dashv\mid \psi$
4. $s \Vdash \forall \mathsf{v}_\mathsf{i} \,.\, \varphi[\mathsf{v}_\mathsf{i}]$ iff there is an $X \subseteq S$ such that
   (a) For all $d \in D$, there is an $s_1 \in X$ such that $s_1 \Vdash \varphi[\mathsf{v}_\mathsf{i} := \mathsf{d}]$, and
   (b) For all $s_1 \in X$, there is a $d \in D$ such that $s_1 \Vdash \varphi[\mathsf{v}_\mathsf{i} := \mathsf{d}]$, and
   (c) $\bigsqcup X$ is defined, and $s = \bigsqcup X$

   and $s \dashv\mid \forall \mathsf{v}_\mathsf{i} \,.\, \varphi[\mathsf{v}_\mathsf{i}]$ iff there is a $d \in D$ such that $s \dashv\mid \varphi[\mathsf{v}_\mathsf{i} := \mathsf{d}]$

Aim

Aim: construct a non-trivial exact truthmaking model $(S, \sqsubseteq, A, |\cdot|; D)$ for a rich enough first order language with predicate symbols S, A, and $\Vdash$ (whose intended reading is *being a state*, *being an actual state*, and *making exactly true* respectively) such that

$$\exists s \in A : s \text{ makes } \varphi \text{ exactly true } \Leftrightarrow$$
$$\exists s \in A : s \text{ makes } (\exists \mathsf{v} \,.\, \mathsf{A}(\mathsf{v}) \wedge \mathsf{v} \Vdash \ulcorner \varphi \urcorner) \text{ exactly true}$$

(Where $\ulcorner \varphi \urcorner$ is a quotation name for the sentence φ).

Why a model for self applicable exact truthmaking?

- A semantics "turning to itself"
- Truth, and hyperintensional contexts and notions
- Paradoxes of truthmaking

A semantic "turning to itself"

Exact truthmaking semantics has recently found applications in the semantics of natural language [16, 17, 18]. And, as [1] (reported in [12]) says, a good semantics for natural language should be able

> *To be turned on itself, and provide an account of its own information content, or rather, of the statements made by the theorist using the theory*

Truth, and hyperintensional contexts and notions

We may want to distinguish between the *hyperintensional content* – (exact truth-conditions) of truth-ascriptions (φ *is true*) and the hyperintensional content of the sentences truth is attributed to (φ).

- **Hyperintensional contexts**. e.g.: The cat knows that the bowl is in the kitchen vs The cat knows that the statement "The bowl is in the kitchen" is true; Alice believes Gödel's 2nd incompleteness theorem is true, because Clara (a logician) told her so, but she doesn't know what Gödel's 2nd incompleteness theorem says [see 24];
- **Aboutness/Subject matter**: "*Sentence 'φ' is true*" is about *a sentence*, φ, while φ may be about something else.

We may consider this difference to be *hyperintensional*, and not *intensional*. Problem: **give a type-free treatment of truth and truthmaking, consistent with the background philosophical picture**.

Truthmaking paradoxes

Study paradoxes of (exact) truthmaking:

- The truth-conditions of this sentence do not obtain, the situation described by this sentence is not actual
- This sentence does not have actual truthmakers
- This sentence does not have truthmakers

For example:

(μ) (μ) does not have actual truthmakers

Suppose it does have actual truthmakers – i.e.: obtaining truth-conditions. Then it is true, and this means that what (μ) says is the case: (μ) does not have actual truthmakers. But this contradicts our assumption. Thus, (μ) does not have actual truthmakers. But this describe precisely the truth-conditions of (μ) itself, and thus (μ) is true. Therefore, (μ) is true, but does not have actual truthmakers.

(Against *truthmaking maximalism*)

Philosophical/semantic preliminary

States: Carnapian *State Description*-like

> *A class of sentences [of $\mathscr{L}$] which contains for every atomic sentence either this sentence or its negation, but not both, and no other sentences, is called a* **state-description** *in [$\mathscr{L}$], because it [...] gives a complete description of a possible sate of the universe of individuals with respect to all properties and relation expressed by predicates of the system. Thus the state descriptions represent Leibinz' possible worlds or Wittgenstein's possible states of affairs [5]*

- Our states will be non-empty sets of literals (i.e.: atomic sentences or negations of atomic sentences) of a language.
- We don't require states to be complete nor consistent.
- $\{P(\vec{t})\} \Vdash P(\vec{t})$, $\{\neg P(\vec{t})\} \dashv\mid P(\vec{t})$[1]; $\sqsubseteq := \subseteq$ restricted to states
- Further requirements later on...

[1] Similar requirements are present in propositional HYPE [see 14], or in the construction of the canonical model of the logic of the exact entailment of [7]

Syntax Theory

- We want to be able to *talk about states*, *about sentences*, and about *ways in which states and sentences of our language are related*. Especially for states and the fact that we will have sentences talking about them: AFA [2] or coding of some kind [8].
- What we need: at least a base structure in which we can effectively code expressions of our language, and in which "syntactical properties, relations, and operations can be reflected" [21].

To be general enough:

- language with denumerable signature, $\mathscr{L}$,
- supported by a *strongly acceptable structure* [19, 8]

A model

Base model: a structure $\mathfrak{M} = (M, R_1, \ldots, R_n)$ such that

- Contains *elementary operations* for coding finite sequences from M;
- There is an elementary ordering of members of M, $\mathcal{N}^{\mathfrak{A}}$, isomorphic to the natural numbers;
- there is an hyperelementary coding of M into $\mathcal{N}^{\mathfrak{A}}$

(In other terms: a *strongly acceptable structure* [see 19, 8])

- We assume $\mathscr{L}$ contains a constant symbol m for every $m \in M$.
- We extend $\mathscr{L}$ to $\mathscr{L}^+$ by adding predicate symbols $\mathsf{S}, \mathsf{A}, \Vdash$ to $\mathscr{L}$. Their intended reading is, respectively: *being a state*, *being an actual state*, *making exactly true*.
- Last requirement on states: we will consider as states *the sets of literals of the expanded language that are "describable" in the structure*.

- From a technical point of view: this restriction will make it possible to *encode our states* in the domain of our structure.
- From a philosophical point of view: captures the intuition that, in formulating a rigorous formal semantics for her language, a person is bound by the mathematical and formal-semantic resources available to her *before* the formulation of the semantics[2].

[2] A similar informal story is present in [8].

Convention:

$\#\varepsilon$ is the object $m \in M$ that encodes the expression ε according to a fixed monotonic coding function $\#$

$\ulcorner\varepsilon\urcorner$ is the closed term of our language denoting $\#\varepsilon$

"Describable sets of literals" – rigorous definition

In [19], Moschovakis shows how to define a satisfaction relation $Sat_{\mathfrak{A}}$, hyperelementary over a base acceptable structure $\mathfrak{A}$, such that $Sat_{\mathfrak{A}}$ bears a code of a formula $\varphi(\mathsf{v_1}, ..., \mathsf{v_n})$ in the first order language of $\mathfrak{A}$ to $(a_1, ..., a_n) \in A^n$ whenever $\mathfrak{A} \models \varphi(\mathsf{v_1}, ..., \mathsf{v_n})[a_1, ..., a_n]$

Definition

A set $X \subseteq M^n$ is **definable in** $\mathfrak{M}$ iff there is a formula $\varphi(\mathsf{v_1}, ..., \mathsf{v_n})$ in the first order language of $\mathfrak{M}$ such that

$$(m_1, ..., m_n) \in X \Leftrightarrow Sat_{\mathfrak{M}}(\#\varphi(\mathsf{v_1}, ..., \mathsf{v_n}), (m_1, ..., m_n))$$

Let $L_{\mathfrak{M}}$ be the set of sets definable in $\mathfrak{M}$. Let now $Form_{\mathscr{L}}$ be the set of codes of formulae of $\mathscr{L}$.
Every set in $L_{\mathfrak{M}}$ is definable by a formula in $\mathscr{L}$. Thus let the (hyperelementary) coding $\pi : L_{\mathfrak{M}} \rightarrow Form_{\mathscr{L}}$ to be

Definition

$$\pi(X) = \mu w \in Form_{\mathscr{L}} . ((m_1, ..., m_n) \in X \Leftrightarrow Sat_{\mathfrak{M}}(w, (m_1, ..., m_n)))$$

This means that we have *objects representing our truth-conditions*, we have *names to refer to them*.

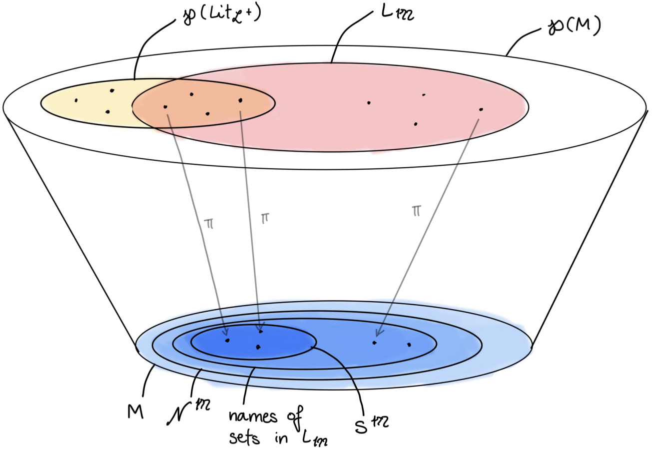
The extension of S

Let $D_\pi \subseteq M$ be the set of codes of objects in $L_{\mathfrak{M}}$, and let $|m|_\pi$ be the object in $L_{\mathfrak{M}}$ associated to $m \in M$ by π.
Then the extensions of our predicates will be defined as follows:

Definition

$\mathsf{S}^{\mathfrak{M}}(x) :\Leftrightarrow D_\pi(x) \;\&\; |x|_\pi \neq \varnothing \;\&\; \forall y \in |x|_\pi \,.\, Lit_{\mathscr{L}^+}(y)$

That is: a *state* is the code of a non-empty set of code of literals of the expanded language.


℘(Lit$_{\mathcal{L}^+}$)
L^{trz}
℘(M)
π
π
π
M
N^{trz}
names of sets in L^{trz}
S^{trz}

The extension of $\Vdash$

Mimics the recursive clauses of $\Vdash$

- $m \Vdash^{\mathfrak{M}} \#\mathsf{P}(\mathsf{t}_1, ..., \mathsf{t}_\mathsf{n})$ iff $m = \pi(\{\#\mathsf{P}(\mathsf{t}_1, ..., \mathsf{t}_\mathsf{n})\})$
- $m \Vdash^{\mathfrak{M}} \#(\varphi \wedge \psi)$ iff $\exists m_1 m_2 \in S^{\mathfrak{M}} . m_1 \Vdash^{\mathfrak{M}} \#\varphi$ and $m_2 \Vdash^{\mathfrak{M}} \#\psi$ and $m = \pi(|m_1|_\pi \cup |m_2|_\pi)$
- $m \Vdash^{\mathfrak{M}} \#\forall \mathsf{v} . \varphi$ iff
  - For any $n \in M$, there is an m_1 that makes $\#\varphi[\mathsf{v} := \mathsf{n}]$ true and $|m_1|_\pi \subseteq |m|_\pi$;
  - For any $c \in Lit_{\mathscr{L}+}$, $c \in |m|_\pi$ only if there's an $n \in M$ such that, for some m_1, $m_1 \Vdash^{\mathfrak{M}} \#\varphi[\mathsf{v} := \mathsf{n}]$ and $c \in |m_1|_\pi$.

The extension of A

A state is a set of literals; an actual state will be a set of *true literals*. Base language: OK – diagram predicate; Extended language?

We need to say which literals of the extended language are true.

Strategy: we will provide a Kripkean truth definition for literals only (an *extended diagram*), and we'll define $A^{\mathfrak{M}}$ using it.

Extended diagram

$\zeta(X, x) :\Leftrightarrow x \in Lit_{\mathscr{L}^+}$, and

- x is the code of a true literal of a base language, or
- x is the code of a true literal of the form $\mathsf{S}(\mathsf{m})$, $\neg\mathsf{S}(\mathsf{m})$, $\mathsf{m}_1 \Vdash \mathsf{m}_2$, or $\mathsf{m}_1 \nVdash \mathsf{m}_2$
- x is a literal of form $\mathsf{A}(\mathsf{m})$, $m \in \mathsf{S}^{\mathfrak{M}}$, and $\forall y \in |m|_\pi \, . \, y \in X$, or
- x is a literal of form $\neg\mathsf{A}(\mathsf{m})$, $m \in \mathsf{S}^{\mathfrak{M}}$, and $\exists y \in |m|_\pi \, . \, Opp(y) \in X$

(Where $Opp : Lit_{\mathscr{L}^+} \to Lit_{\mathscr{L}^+}$ outputs $\dot{\neg} x$ if $x \in Atom_{\mathscr{L}}$, and erases $\neg$ in front of x if x is the code of a negation of an atomic sentence).

The operator Γ_ζ

Definition

$$\Gamma^\alpha_\zeta(X) = \begin{cases} X, & \text{if } \alpha = 0 \\ \{m \in M : \zeta(\Gamma^{\alpha-1}_\zeta(X), m)\}, & \text{if } \alpha \text{ is a successor ordinal} \\ \bigcup_{\beta<\alpha} \Gamma^\beta_\zeta(X), & \text{if } \alpha \text{ is a limit ordinal} \end{cases}$$

Observation

Γ *is monotonic: if* $X \subseteq Y$, *then, for every* $\alpha \in On$, $\Gamma^\alpha_\zeta(X) \subseteq \Gamma^\alpha_\zeta(Y)$

Observation

There is a $\beta \in On$ *such that* $\Gamma^\beta_\zeta(\varnothing) = \Gamma^{\beta+1}_\zeta(\varnothing)$

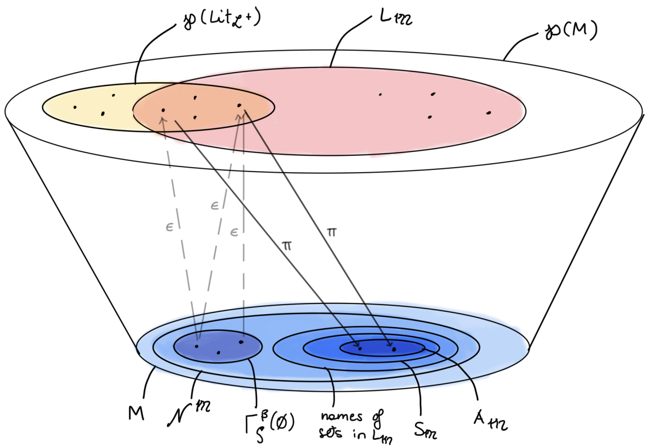
The extension of A

Let β be the minimal ordinal such that $\Gamma_\zeta^\beta(\varnothing) = \Gamma_\zeta^{\beta+1}(\varnothing)$. Then $\Gamma_\zeta^\beta(\varnothing)$ is the set of true literals we were looking for. Then let

Definition

$\mathsf{A}^{\mathfrak{M}}(x) :\Leftrightarrow \mathsf{S}^{\mathfrak{M}}(x) \;\&\; \forall y \in |x|_\pi \,.\, y \in \Gamma_\zeta^\beta(\varnothing)$

That is, an object $m \in M$ is an the extension of A iff it is the code of a state and the state encoded by it contains only codes of true literals.



℘(Lit_L+)
L_m
℘(M)
ϵ
ϵ
ϵ
π
π
M
N^m
Γ_S^β(∅)
names of sets in L_m
S_m
A_m

Facts

The following facts are provable about our construction:

Fact (1)

Let φ be a formula of the language of $\mathfrak{M}$. Then φ is true in $\mathfrak{M}$ iff there is an $m \in \mathsf{A}^{\mathfrak{M}}$ such that $m \Vdash^{\mathfrak{M}} \#\varphi$.
In other terms: $\lambda\xi \in Sent_{\mathscr{L}} \,.\, \exists \mathsf{v} \,.\, \mathsf{A}(\mathsf{v}) \wedge \mathsf{v} \Vdash \ulcorner\xi\urcorner$ is a truth-predicate for $\mathscr{L}$ interpreted in $\mathfrak{M}$.

Fact (2)

Let φ be a formula of the extended language. Then it is true in $(\mathfrak{M}, \mathsf{S}^{\mathfrak{M}}, \mathsf{A}^{\mathfrak{M}}, \Vdash^{\mathfrak{M}})$ that there is an $m \in \mathsf{A}^{\mathfrak{M}}$ such that $m \Vdash^{\mathfrak{M}} \#\varphi$ iff there is an $m \in \mathsf{A}^{\mathfrak{M}}$ such that $m \Vdash^{\mathfrak{M}} \#(\exists \mathsf{v} \,.\, \mathsf{A}(\mathsf{v}) \wedge \mathsf{v} \Vdash \ulcorner\varphi\urcorner)$.

Fact (3)

Let

- $S := \{x \in L_{\mathfrak{M}} : \mathsf{S}^{\mathfrak{M}}(\pi(x))\}$
- $\sqsubseteq := \subseteq\restriction S$
- $A := \{x \in L_{\mathfrak{M}} : \mathsf{A}^{\mathfrak{M}}(\pi(x))\}$
- $|\cdot| := \{(\varphi, (\{\{\varphi\}\}, \{\{\neg\varphi\}\}))$: φ *is an atomic sentence of* $\mathscr{L}^+\}$
- $D := M$

Then $(S, \sqsubseteq, A, |\cdot|; D)$ *is an exact truthmaking model for* $\mathscr{L}^+$*, and the model is such that:*

- *for any sentence* φ *of* $\mathscr{L}$*, there's a state* $s \in S \cap A$ *making* φ *exactly true iff* $\mathfrak{M} \models \varphi$*, and*
- *for any sentence* φ *of* $\mathscr{L}^+$*, there's a state* $s \in S \cap A$ *making* φ *exactly true iff there's a state* $s \in S \cap A$ *making* $\exists \mathsf{v}\,.\,\mathsf{A}(\mathsf{v}) \land \mathsf{v} \Vdash \ulcorner\varphi\urcorner$ *exactly true.*

What is in the making, and what to do next

- Tackle revenge: repeat the construction on the extended structure, produce a new exact truthmaking model with new predicates $\rightsquigarrow$ a hierarchy of exact truthmaking models, each of them capturing more obtaining truth-conditions [8].
- Add a predicate P for *possible states*. Rough idea: extend the framework of Halbach Welch and Stern [10, 23].
- We could have used $\mathbb{H}YP_{\mathfrak{M}}$-sets of literals: see what happens. Also: see what happens with generalized quantifiers.
- Modelling hyperintensional contexts, and study aboutness of truth-claims.
- Theory of facts : semantic exact truthmaking $=$ correspondence theory of truth : semantic notion of truth?

So long, and thanks for all the fish!

[1] Jon Barwise. *The situation in logic*, volume 4. Center for the Study of Language (CSLI), 1989.

[2] Jon Barwise and John Etchemendy. *The liar: An essay on truth and circularity*. Oxford University Press, 1987.

[3] Jon Barwise and John Perry. Situations and attitudes. *The Journal of Philosophy*, 78(11):668–691, 1981.

[4] Tim Button and Sean Walsh. *Philosophy and model theory*. Oxford University Press, 2018.

[5] Rudolf Carnap. *Meaning and Necessity: a Study in Semantics and Modal Logic*. University of Chicago Press, 1947.

[6] Kit Fine. Truthmaker semantics. *A Companion to the Philosophy of Language*, 2:556–577, 2017.

[7] Kit Fine and Mark Jago. Logic for exact entailment. *The Review of Symbolic Logic*, 12(3):536–556, 2019.

[8] Michael Glanzberg. A contextual-hierarchical approach to truth and the liar paradox. *Journal of Philosophical Logic*, 33(1):27–88, 2004.

[9] Michael Glanzberg. Quantification and realism. *Philosophy and Phenomenological Research*, 69(3):541–572, 2004.

[10] Volker Halbach, Hannes Leitgeb, and Philip Welch. Possible-worlds semantics for modal notions conceived as predicates. *Journal of Philosophical Logic*, 32(2):179–223, 2003.

[11] Angelika Kratzer. An investigation of the lumps of thought. *Linguistics and philosophy*, 12(5):607–653, 1989.

[12] Angelika Kratzer. Situations in Natural Language Semantics. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2020 edition, 2020.

[13] Saul A Kripke. Semantical analysis of intuitionistic logic i. In *Studies in Logic and the Foundations of Mathematics*, volume 40, pages 92–130. Elsevier, 1965.

[14] Hannes Leitgeb. Hype: A system of hyperintensional logic (with an application to semantic paradoxes). *Journal of Philosophical Logic*, 48(2):305–405, 2019.

[15] David Lewis. General semantics. In *Montague grammar*, pages 1–50. Elsevier, 1976.

[16] Friederike Moltmann. Cognitive products and the semantics of attitude verbs and deontic modals. *Act-based conceptions of propositional content*, pages 254–290, 2017.

[17] Friederike Moltmann. Truth predicates, truth bearers, and their variants. *Synthese*, pages 1–28, 2018.

[18] Friederike Moltmann. Truthmaker-based content: Syntactic, semantic, and ontological contexts. *Theoretical Linguistics5*, 2021.

[19] Yiannis N Moschovakis. *Elementary induction on abstract structures*. North-Holland Publishing Company, 1974.

[20] Ilkka Niiniluoto. Tarski's definition and truth-makers. *Annals of Pure and Applied Logic*, 126(1-3):57–76, 2004.

[21] Panu Raatikainen. Gödel's Incompleteness Theorems. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Spring 2022 edition, 2022.

[22] Bertrand Russell and Bertrand Russell. *Logical atomism*. Fontana London, 1972.

[23] Johannes Stern. *Toward predicate approaches to modality*, volume 44. Springer, 2015.

[24] Johannes Stern. Belief, truth, and ways of believing. In *Modes of Truth*, pages 151–181. Routledge, 2021.

[25] Neil Tennant and M Glanzberg. A logical theory of truthmakers and falsitymakers. *The Oxford Handbook of Truth*, page 355, 2018.

[26] Bas C Van Fraassen. Facts and tautological entailments. *The Journal of Philosophy*, 66(15):477–487, 1969.

[27] Ludwig Wittgenstein. *Tractatus Logico-Philosophicus: Centenary Edition*. Anthem Press, 2021.

Recursive Clauses

Definition

1. $s \Vdash \mathsf{P}(\mathsf{t_1}, ..., \mathsf{t_n})$ iff $s \in proj_1|\mathsf{P}(\mathsf{t_1}, ..., \mathsf{t_n})|$, and
   $s \dashv\vdash \mathsf{P}(\mathsf{t_1}, ..., \mathsf{t_n})$ iff $s \in proj_2|\mathsf{P}(\mathsf{t_1}, ..., \mathsf{t_n})|$
2. $s \Vdash \neg\varphi$ iff $s \dashv\vdash \varphi$, and $s \dashv\vdash \neg\varphi$ iff $s \Vdash \varphi$
3. $s \Vdash \varphi \wedge \psi$ iff there are $s_1, s_2 \in S$ such that
   $s_1 \Vdash \varphi \ \& \ s_2 \Vdash \psi \ \& \ s_1 \sqcup s_2$ is defined, and $s = s_1 \sqcup s_2$, and
   $s \dashv\vdash \varphi \wedge \psi$ iff $s \dashv\vdash \varphi$ or $s \dashv\vdash \psi$
4. $s \Vdash \forall \mathsf{v_i} \,.\, \varphi[\mathsf{v_i}]$ iff there is an $X \subseteq S$ such that
   (a) For all $d \in D$, there is an $s_1 \in X$ such that $s_1 \Vdash \varphi[\mathsf{v_i} := \mathsf{d}]$, and
   (b) For all $s_1 \in X$, there is a $d \in D$ such that $s_1 \Vdash \varphi[\mathsf{v_i} := \mathsf{d}]$, and
   (c) $\bigsqcup X$ is defined, and $s = \bigsqcup X$

   and $s \dashv\vdash \forall \mathsf{v_i} \,.\, \varphi[\mathsf{v_i}]$ iff there is a $d \in D$ such that $s \dashv\vdash \varphi[\mathsf{v_i} := \mathsf{d}]$

Definition (Exact truthmaking – Part 1)

$x \Vdash^{\mathfrak{M}} y :\Leftrightarrow \mathsf{S}^{\mathfrak{M}}(x) \ \& \ Sent_{\mathscr{L}+}(y) \ \&$

- $\exists m_1 ... m_n \in CTerm_{\mathscr{L}+} \,.\, y = \#\mathsf{P}(\mathsf{m}_1, ..., \mathsf{m}_n) \ \& \ x = \pi(\{y\})$, or
- $\exists m_1 ... m_n \in CTerm_{\mathscr{L}+} \,.\, y = \#\neg\mathsf{P}(\mathsf{m}_1, ..., \mathsf{m}_n) \ \& \ x = \pi(\{y\})$, or
- $\exists w_1 \in Sent_{\mathscr{L}+} \,.\, y = \dot{\neg}\dot{\neg} w_1 \ \& \ x \Vdash^{\mathfrak{M}} w_1$
- $\exists w_1 w_2 \in Sent_{\mathscr{L}+} \,.\, y = w_1 \dot{\wedge} w_2 \ \&$
  $\exists x_1 x_2 \in \mathsf{S}^{\mathfrak{M}} \,.\, x_1 \Vdash^{\mathfrak{M}} w_1 \ \& \ x_2 \Vdash^{\mathfrak{M}} w_2 \ \& \ x = \pi(|x_1|_\pi \cup |x_2|_\pi)$, or
- $\exists w_1 w_2 \in Sent_{\mathscr{L}+} \,.\, y = \dot{\neg}(w_1 \dot{\wedge} w_2) \ \& \ x \Vdash^{\mathfrak{M}} \dot{\neg} w_1$ or $x \Vdash^{\mathfrak{M}} \dot{\neg} w_2$, or
- $\exists w_1 w_2 \in Sent_{\mathscr{L}+} \,.\, y = w_1 \dot{\vee} w_2 \ \& \ x \Vdash^{\mathfrak{M}} w_1$ or $x \Vdash^{\mathfrak{M}} w_2$, or
- $\exists w_1 w_2 \in Sent_{\mathscr{L}+} \,.\, y = \dot{\neg}(w_1 \dot{\vee} w_2) \ \&$
  $\exists x_1 x_2 \in \mathsf{S}^{\mathfrak{M}} \,.\, x_1 \Vdash^{\mathfrak{M}} \dot{\neg} w_1 \ \& \ x_2 \Vdash^{\mathfrak{M}} \dot{\neg} w_2 \ \& \ x = \pi(|x_1|_\pi \cup |x_2|_\pi)$, or

Definition (Exact truthmaking – Part 2)

- $\exists v_1 \in Var_{\mathscr{L}+} . \exists w_1 \in Form_{\mathscr{L}+} . y = \dot{\forall} v_1 w_1$ &
  - (a) $\forall m_1 \in CTerm_{\mathscr{L}+} . \exists z \in \mathsf{S}^{\mathfrak{M}} . |z|_\pi \subseteq |x|_\pi$ & $\mathbf{z} \Vdash^{\mathfrak{M}} w_1[v_1 := m_1]$, &
  - (b) $\forall u \in |x|_\pi . \exists m_1 \in CTerm_{\mathscr{L}+} . \exists z \in \mathsf{S}^{\mathfrak{M}} . |z|_\pi \subseteq |x|_\pi$ & $z \Vdash^{\mathfrak{M}} w_1[v_1 := m_1]$ & $u \in |z|_\pi$
- $\exists v_1 \in Var_{\mathscr{L}+} . \exists w_1 \in Form_{\mathscr{L}+} . y = \dot{\neg}\dot{\forall} v_1 w_1$ & $\exists m_1 \in CTerm_{\mathscr{L}+} . x \Vdash^{\mathfrak{M}} \dot{\neg} w_1[v_1 := m1]$
- $\exists v_1 \in Var_{\mathscr{L}+} . \exists w_1 \in Form_{\mathscr{L}+} . y = \dot{\exists} v_1 w_1$ & $\exists m_1 \in CTerm_{\mathscr{L}+} . x \Vdash^{\mathfrak{M}} w_1[v_1 := m1]$
- $\exists v_1 \in Var_{\mathscr{L}+} . \exists w_1 \in Form_{\mathscr{L}+} . y = \dot{\neg}\dot{\exists} v_1 w_1$ &
  - (a) $\forall m_1 \in CTerm_{\mathscr{L}+} . \exists z \in \mathsf{S}^{\mathfrak{M}} . |z|_\pi \subseteq |x|_\pi$ & $\mathbf{z} \Vdash^{\mathfrak{M}} \dot{\neg} w_1[v_1 := m_1]$, &
  - (b) $\forall u \in |x|_\pi . \exists m_1 \in CTerm_{\mathscr{L}+} . \exists z \in \mathsf{S}^{\mathfrak{M}} . |z|_\pi \subseteq |x|_\pi$ & $z \Vdash^{\mathfrak{M}} \dot{\neg} w_1[v_1 := m_1]$ & $u \in |z|_\pi$

The clauses for quantifiers are related to the ones expressed before by the following proposition:

Proposition (Proposition 1)

Suppose $\mathscr{K}$ *is the first order language of a strongly acceptable structure* $\mathfrak{M}$ *augmented with a constant* m *for each* $m \in M$*. Suppose our set of states,* S*, is the set of literals of* $\mathscr{K}$*, and the parthood relation* $\sqsubseteq$ *is just* $\subseteq\restriction S$*. Then:*

$$(\exists X \,.\, (\forall m \in M \,.\, \exists x \in X \,.\, x \Vdash \varphi[\mathsf{v_i} := \mathsf{m}])\wedge$$
$$(\forall x \in X \,.\, \exists m \in M \,.\, x \Vdash \varphi[\mathsf{v_i} := \mathsf{m}])\wedge$$
$$s = l.u.b._{\sqsubseteq} X) \Leftrightarrow$$

$$((\forall m \in M \,.\, \exists r \subseteq s \,.\, r \Vdash \varphi[\mathsf{v_i} := \mathsf{m}])\wedge$$
$$(\forall u \in Lit_{\mathscr{K}} . u \in s \Rightarrow (m \in M \,.\, \exists r \subseteq s \,.\, r \Vdash$$
$$\varphi[\mathsf{v_i} := \mathsf{m}] \wedge u \in r)))$$

Let $Opp : Lit_{\mathscr{L}^+} \to Lit_{\mathscr{L}^+}$ be the function that, for any $x \in Lit_{\mathscr{L}^+}$, outputs the code of the negation of x if x is the code of an atomic sentence, and the atomic sentence that results from erasing the negation in front of x if x is the negation of an atomic sentence. Furthermore, let $Val : M \to M$ be the function that sends the code of a closed term to the object in M the term denotes, and let the formula $\zeta(X, x)$ be the following:

Definition

$\zeta(X, x) :\Leftrightarrow Lit_{\mathscr{L}+}(x)$ &

1. $Lit_{\mathscr{L}}(x)$ & $\delta_{\mathfrak{M}}(x)$, or
2. $\exists m_1 \in CTerm_{\mathscr{L}+} . x = \dot{\mathsf{S}}(m_1)$ & $\mathsf{S}^{\mathfrak{M}}(Val(m_1))$, or
3. $\exists m_1 \in CTerm_{\mathscr{L}+} . x = \dot{\neg}\dot{\mathsf{S}}(m_1)$ & $\neg\mathsf{S}^{\mathfrak{M}}(Val(m_1))$, or
4. $\exists m_1 \in CTerm_{\mathscr{L}+} . x = \dot{\mathsf{A}}(m_1)$ & $\forall u \in |Val(m_1)|_\pi . X(u)$, or
5. $\exists m_1 \in CTerm_{\mathscr{L}+} . x = \dot{\neg}\dot{\mathsf{A}}(m_1)$ & $\exists u \in |Val(m_1)|_\pi . X(Opp(u))$, or
6. $\exists m_1 m_2 \in CTerm_{\mathscr{L}+} . x = m_1 \dot{\Vdash} m_2$ & $Val(m_1) \Vdash^{\mathfrak{M}} Val(m_2)$, or
7. $\exists m_1 m_2 \in CTerm_{\mathscr{L}+} . x = m_1 \dot{\nVdash} m_2$ & $Val(m_1) \nVdash^{\mathfrak{M}} Val(m_2)$

Sketch of proof of Fact 1

The proposition is proved by induction on the complexity of the sentences. The difficult part is with the universal quantifiers. One direction is basically proved using an $\mathfrak{M} - rule$; as for the other direction, we need a little detour. The induction hypothesis is sufficient to prove every clause, apart form the one of the universal quantification: indeed, suppose we have an actual truthmaker for every instance of a universally quantified sentence. We need to show that there is an actual truthmaker for the universally quantified sentence. But we have a limitation: *definability*.

The following predicate is definable in the language

Definition (Sub-literals)

$\mathsf{SubLit}(w,x) \leftrightarrow \mathsf{Sent}_{\mathscr{L}}(w) \wedge \mathsf{Lit}_{\mathscr{L}}(x) \wedge$

1. $(\mathsf{Lit}_{\mathscr{L}}(w) \wedge x = w) \vee$
2. $(\exists w_1 w_2 \in \mathsf{Sent}_{\mathscr{L}} . (w = w_1 \dot{\wedge} w_2 \vee w = w_1 \dot{\vee} w_2) \wedge$
   $(\mathsf{SubLit}(w_1, x) \vee \mathsf{SubLit}(w_2, x))) \vee$
3. $(\exists w_1 w_2 \in \mathsf{Sent}_{\mathscr{L}} . (w = \dot{\neg}(w_1 \dot{\wedge} w_2) \vee w = \dot{\neg}(w_1 \dot{\vee} w_2)) \wedge$
   $(\mathsf{SubLit}(\dot{\neg} w_1, x) \vee \mathsf{SubLit}(\dot{\neg} w_2, x))) \vee$
4. $(\exists w_1 \in \mathsf{Sent}_{\mathscr{L}} . w = \dot{\neg}\dot{\neg} w_1 \wedge \mathsf{SubLit}(w_1, x)) \vee$
5. $(\exists v_1 \in \mathsf{Var} . \exists w_1 \in \mathsf{Form}_{\mathscr{L}} . (w = \dot{\forall} v_1 w_1 \vee w = \dot{\exists} v_1 w_1) \wedge$
   $\exists m \in \mathsf{CTerm}_{\mathscr{L}} . \mathsf{SubLit}(w_1[v_1 := m, x)) \vee$
6. $(\exists v_1 \in \mathsf{Var} . \exists w_1 \in \mathsf{Form}_{\mathscr{L}} . (w = \dot{\neg}\dot{\forall} v_1 w_1 \vee w = \dot{\neg}\dot{\exists} v_1 w_1) \wedge$
   $\exists m \in \mathsf{CTerm}_{\mathscr{L}} . \mathsf{SubLit}(\dot{\neg} w_1[v_1 := m, x))$

So, the set defined by $\mathsf{SubLit}(\ulcorner\varphi\urcorner, \mathsf{v_i})$ exists in $L_{\mathfrak{M}}$. Furthermore, the set of true literals of the base language is definable in the language (and is elementary) *via* the diagram predicate ($\delta_{\mathfrak{M}}$). The result for universally quantified sentences follows by considering the predicate

$$\mathsf{Act}(\ulcorner\varphi\urcorner, \mathsf{x}) \leftrightarrow \mathsf{SubLit}(\ulcorner\varphi\urcorner, \mathsf{x}) \wedge \delta_{\mathfrak{M}}(\mathsf{x})$$

Proof of Fact 2

Suppose

$$\mathfrak{M}^{+}, \sigma \vDash \exists \mathsf{x} \,.\, \mathsf{A}(\mathsf{x}) \;\wedge\; \mathsf{x} \Vdash \ulcorner \exists \mathsf{v} \,.\, \mathsf{A}(\mathsf{v}) \;\wedge\; \mathsf{v} \Vdash \ulcorner \varphi \urcorner \urcorner$$

This is true iff there is an $m \in M$ such that $m \in \mathsf{A}^{\mathfrak{M}}$ and $m \Vdash^{\mathfrak{M}} \# \exists \mathsf{v} \,.\, \mathsf{A}(\mathsf{v}) \;\wedge\; \mathsf{v} \Vdash \ulcorner \varphi \urcorner$. By recursive clauses of the definition of $\Vdash^{\mathfrak{M}}$, we know that the second conjunct is the case iff there is a closed term t such that

$$m \Vdash^{\mathfrak{M}} \#(\mathsf{A}(\mathsf{t}) \;\wedge\; \mathsf{t} \Vdash \ulcorner \varphi \urcorner)$$

Which, in turn, is the case iff there are m_1 and m_2 in $\mathsf{S}^{\mathfrak{M}}$ such that

$$m_1 \Vdash^{\mathfrak{M}} \# \mathsf{A}(\mathsf{t}),\; m_2 \Vdash^{\mathfrak{M}} \# \mathsf{t} \Vdash \ulcorner \varphi \urcorner,\; \text{and } m = \pi(|m_1|_\pi \cup |m_2|_\pi)$$

Since A(t) and t $\Vdash \ulcorner\varphi\urcorner$ are atomic formulae, we know that their exact truthmakers are the codes of their singletons. Furthermore, since m is actual, then both m_1 and m_2 must be actual – that is: $m_1 \in \mathsf{A}^{\mathfrak{M}}$ and $m_2 \in \mathsf{A}^{\mathfrak{M}}$ (notice that also the converse holds: if m_1 and m_2 are actual, then their union is). But this means that

$$Den(\mathsf{t}) \in \mathsf{A}^{\mathfrak{M}} \text{ and } Den(\mathsf{t}) \Vdash^{\mathfrak{M}} Den(\ulcorner\varphi\urcorner)$$

Which is the case iff

$$\mathfrak{M}^{+}, \sigma \vDash \exists \mathsf{x} \,.\, \mathsf{A}(\mathsf{x}) \;\wedge\; \mathsf{x} \Vdash \ulcorner\varphi\urcorner$$

Paradoxes

It is not the case that, for every X, $\Gamma_\zeta(X)$ reaches a *sound* fixed point.
Suppose that an operation of strong diagonalization is available. Then there is a term l such that $l = \pi(\{\#\neg\mathsf{A}(\mathsf{l})\})$. Let l be such that $l = Den(\mathsf{l})$. Suppose now that $\#\mathsf{A}(\mathsf{l}) \in X$. Informally speaking, we could think of l as the name of a state that is *fittingly described* by the sentence $\neg\mathsf{A}(\mathsf{l})$, i.e. *the state l is non actual*, or *the truth-conditions of this sentence do not obtain*. Thus, the state description whose name is l is an actual truth-condition iff the state description described by l is non actual.

This is represented in our system by the fact that

$$l \Vdash^{\mathfrak{M}} \#\neg \mathsf{A}(\mathsf{l})$$

By hypothesis, we know that $\#\mathsf{A}(\mathsf{l}) \in \Gamma^0_\zeta(X)$. Since l is a name of $\mathscr{L}^+$, $l \in \mathsf{S}^{\mathfrak{M}}$, and $\#\neg\mathsf{A}(\mathsf{l}) \in |Den(\mathsf{l})|_\pi \;\&\; Opp(\#\neg\mathsf{A}(\mathsf{l})) \in \Gamma^0_\zeta(X)$, then we have that $\#\neg\mathsf{A}(\mathsf{l}) \in \Gamma^1_\zeta(X)$ by definition of Γ_ζ, and we also know that $\#\mathsf{A}(\mathsf{l}) \notin \Gamma^1_\zeta(X)$. Thus, there are arguments that *flip-flop*, and thus that are not in any sound fixed point of Γ_ζ.

What about the sentence (μ)? As for μ, it can be formalized in our language $\mathscr{L}^+$ as follows:

$$\forall \mathsf{v}\,.\,\mathsf{A}(\mathsf{v}) \rightarrow \mathsf{v} \nVdash \ulcorner \mu \urcorner$$

We can see that $\mathfrak{M}^+ \models \mu$, but, on pain of internal inconsistency of the predicate A (but consider that we are using the minimal fixed point/sound fixed points of the operator Γ to fix its extension), there is no $m \in \mathsf{A}^{\mathfrak{M}}$ such that $m \Vdash^{\mathfrak{M}} \# \mu$. Thus the paradox is present here... in a sense.

However, since there is no $m \in A^{\mathfrak{M}}$ such that $m \Vdash^{\mathfrak{M}} \#\mu$, (μ) *is not true in our exact truthmaking model* (the one described in Fact 3). That is:

$$\neg\exists s \in A \,.\, s \Vdash \mu$$

So, in a sense, while the paradox is present *in our supporting structure*, it is not present *in our target model*, which is the important result.

- Being true in the supporting structure vs being true in the target model: the target model doesn't capture *all the facts* of the supporting structure.
- It is possible to repeat our construction over $\mathfrak{M}^+$ to capture the revenge intuition – this is close to a *contextual approach* to paradoxes.

Davidson on the Great Fact

$$
\begin{array}{l}
\forall\varphi\psi\,.\,(\forall\mathfrak{M}\in Mod\,.\,\mathfrak{M}\models\varphi\Leftrightarrow\mathfrak{M}\models\psi)\Rightarrow\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[\varphi])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\psi])\\
\forall t_1 t_2\varphi\,.\,\mathfrak{A}\models t_1\doteq t_2\Rightarrow\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[\varphi[\mathsf{v}:=t_1]])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\varphi[\mathsf{v}:=t_2]])\\
\forall\mathfrak{M}\in Mod\,.\,\mathfrak{M}\models(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\varphi)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a})\Leftrightarrow\mathfrak{M}\models\varphi\\
\forall\varphi\,.\,\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\varphi)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a})])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\varphi])\\
\quad\mathfrak{A}\models\vartheta_1\\
\quad\quad\mathfrak{A}\models\vartheta_2\\
\quad\quad\mathfrak{A}\models(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\vartheta_1)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\vartheta_2)\\
\quad\quad\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\vartheta_1)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a})])\leftrightarrow \mathsf{Corr}(\mathsf{x},[(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\vartheta_2)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a})])\\
\quad\quad\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\vartheta_1)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a})])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\vartheta_1])\\
\quad\quad\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a}\wedge\vartheta_2)\doteq(\iota \mathsf{v}\,.\,\mathsf{v}\doteq \mathsf{a})])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\vartheta_2])\\
\quad\quad\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[\vartheta_1])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\vartheta_2])\\
\quad\mathfrak{A}\models\vartheta_2\Rightarrow\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[\vartheta_1])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\vartheta_2])\\
\mathfrak{A}\models\vartheta_1\Rightarrow(\mathfrak{A}\models\vartheta_2\Rightarrow\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[\vartheta_1])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\vartheta_2]))\\
\forall\varphi\psi\,.\,\mathfrak{A}\models\varphi\Rightarrow(\mathfrak{A}\models\psi\Rightarrow\mathfrak{A}\models\forall \mathsf{x}\in \mathsf{Sent}\,.\,\mathsf{Corr}(\mathsf{x},[\varphi])\leftrightarrow \mathsf{Corr}(\mathsf{x},[\psi]))\\
\forall\varphi\,.\,\mathfrak{A}\models\varphi\Rightarrow\mathfrak{A}\models \mathsf{Corr}(\ulcorner\varphi\urcorner,[\varphi])\\
\forall\varphi\psi\,.\,\mathfrak{A}\models\psi\Rightarrow\mathfrak{A}\models \mathsf{Corr}(\ulcorner\varphi\urcorner,[\psi])
\end{array}
$$

A very humble notion of fact

Extend the language with an *epsilon operator* ε, and extend the model so to have an interpretation for a language containing it – canonically: $(\mathfrak{M}^+, \Phi)$, where $\Phi : \wp(M) \to M$ is a choice function. We need an hyperintensional notion. Proposal

- $[\varphi] := \varepsilon \mathsf{u} \, . \, \mathsf{A}(\mathsf{u}) \wedge \mathsf{u} \Vdash \ulcorner \varphi \urcorner$
- $\mathsf{Corr}(\mathsf{x}, \mathsf{y}) \Leftrightarrow \mathsf{Sent}(\mathsf{x}) \wedge \mathsf{S}(\mathsf{y}) \wedge \mathsf{A}(\mathsf{y}) \wedge \mathsf{y} \Vdash \mathsf{x}$

Then we can see that

- Hyperintensionality comes from the hyperintensional context $[-]$, which in turn is defined in terms of the hyperintensional context $\Vdash$;
- Given basic rules of ε-calculus and our philosophical considerations, it is the case that

$$(\mathfrak{M}^+, \Phi) \models \varphi \Leftrightarrow (\mathfrak{M}^+, \Phi) \models \exists \mathsf{x}\,.\, \mathsf{Corr}(\ulcorner\varphi\urcorner, \mathsf{x})$$

- Suppose φ is true. Then there is an x such that
$(\mathfrak{M}^+, \Phi) \models \mathsf{x} \Vdash \ulcorner\varphi\urcorner \rightsquigarrow \varepsilon$-calculus:
$(\mathfrak{M}^+, \Phi) \models [\varphi] \in \mathsf{A} \wedge [\varphi] \Vdash \ulcorner\varphi\urcorner \rightsquigarrow (\mathfrak{M}^+, \Phi) \models \mathsf{Corr}(\ulcorner\varphi\urcorner, [\varphi])$