Reasoning about Natural Language: (Monoids, Tensors)

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QMUL => University College London

Natural Language

Natural languages have different characteristic features and computational linguists look for mathematical structures that model and reason about them.

Characteristic Features

Morphology, phenology, syntax, semantics, pragmatics, speech recognition, computational complexity, parsing, translation, . …

Rules of Grammar

Patterns of Meaning

Characteristic Features

Morphology, phenology, syntax, semantics, pragmatics, speech recognition, computational complexity, parsing, translation, . …

Rules of Grammar — Syntax

Patterns of Meaning — Semantics

Rules of Grammar

Rules of Grammar tell us how to put the words of a language together to form sentences.

I have a car.

x have a I car.

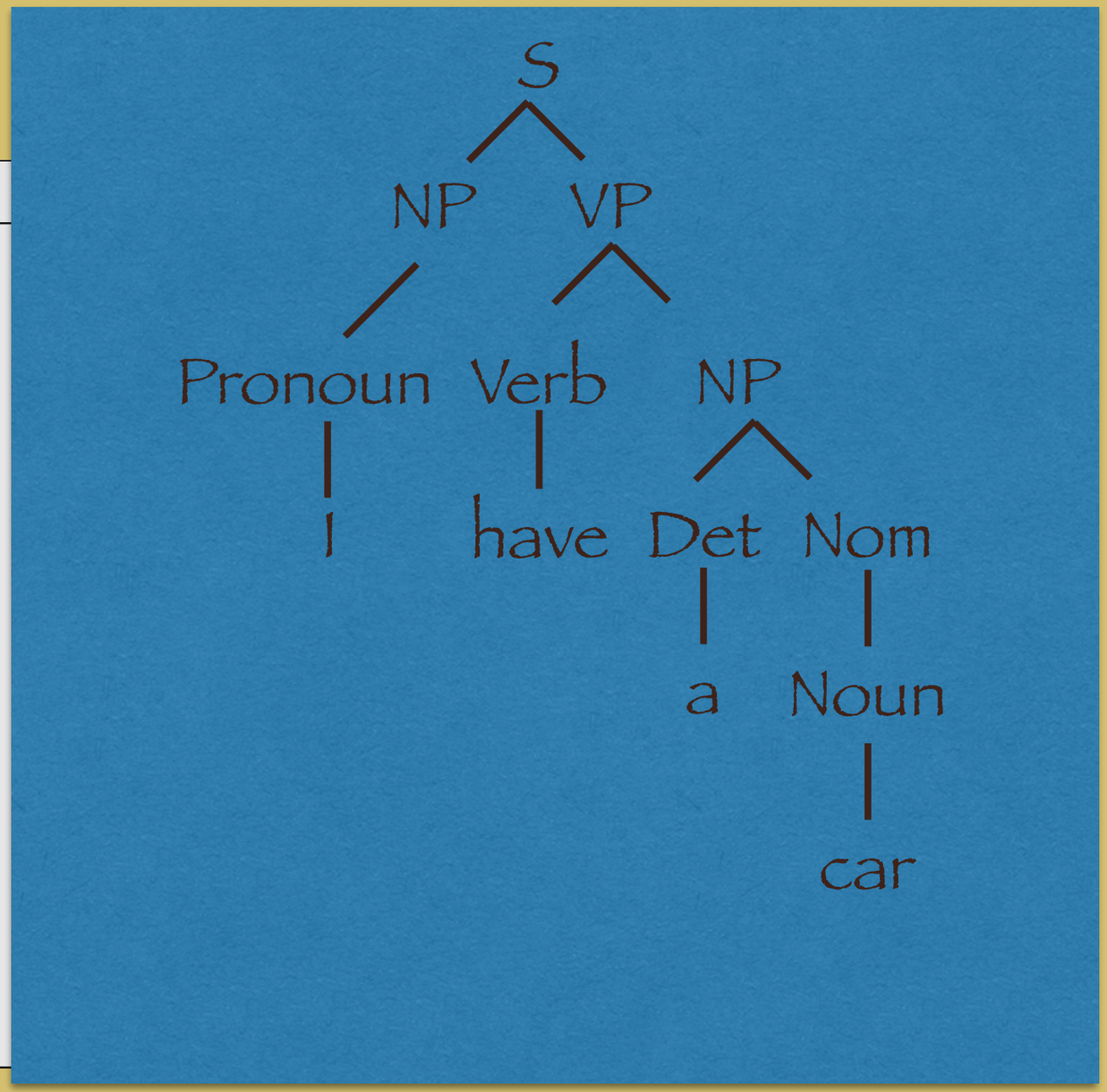
x I car a have.

Examples

Context Free Grammar

$$(N, \Sigma, R, S)$$

Grammar Rules		
S	$\rightarrow$	$NP\ VP$
NP	$\rightarrow$	$Pronoun$
	$\mid$	$Proper\text{-}Noun$
	$\mid$	$Det\ Nominal$
$Nominal$	$\rightarrow$	$Nominal\ Noun$
	$\mid$	$Noun$
VP	$\rightarrow$	$Verb$
	$\mid$	$Verb\ NP$
	$\mid$	$Verb\ NP\ PP$
	$\mid$	$Verb\ PP$
PP	$\rightarrow$	$Preposition\ NP$



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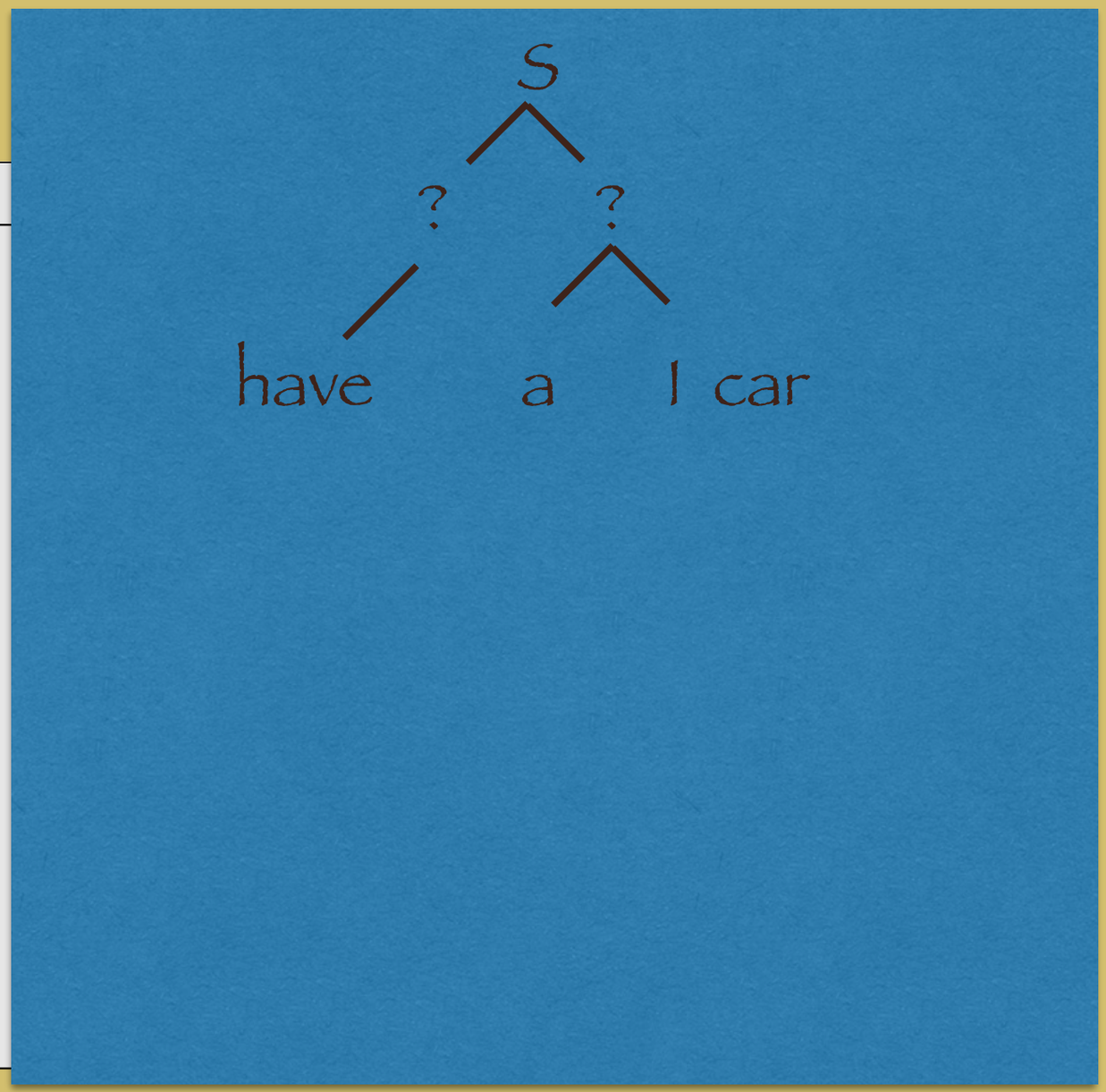
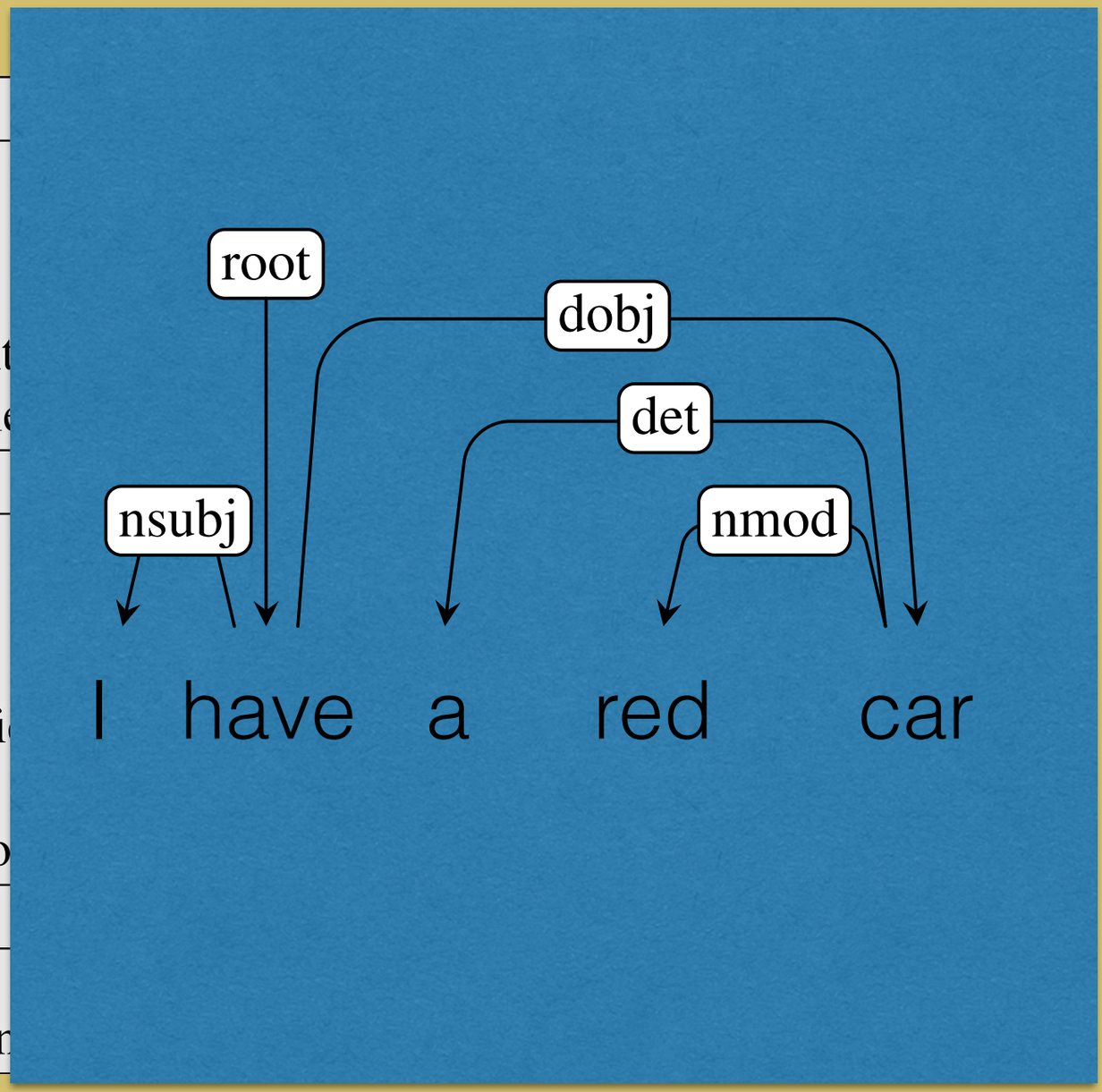


Example

Dependancy Graphs
G = (V, A, L)

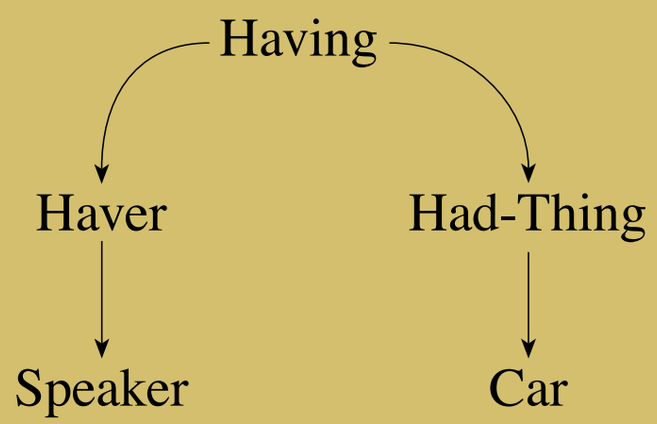
Clausal Argument Relations	Description
NSUBJ	Nominal subject
DOBJ	Direct object
IOBJ	Indirect object
CCOMP	Clausal complement
XCOMP	Open clausal comple
Nominal Modifier Relations	**Description**
NMOD	Nominal modifier
AMOD	Adjectival modifier
NUMMOD	Numeric modifier
APPOS	Appositional modifi
DET	Determiner
CASE	Prepositions, postpo
Other Notable Relations	**Description**
CONJ	Conjunct
CC	Coordinating conjun



Patterns of Meaning

Words have different meanings attached to them, these get composed to form a meaning for sentences that contain them.

I have a car.



Examples

AMR

Car

⇑ POSS-BY

Speaker

Record Semantics

```
Having
 Haver:   Speaker
 HadThing:  Car
```

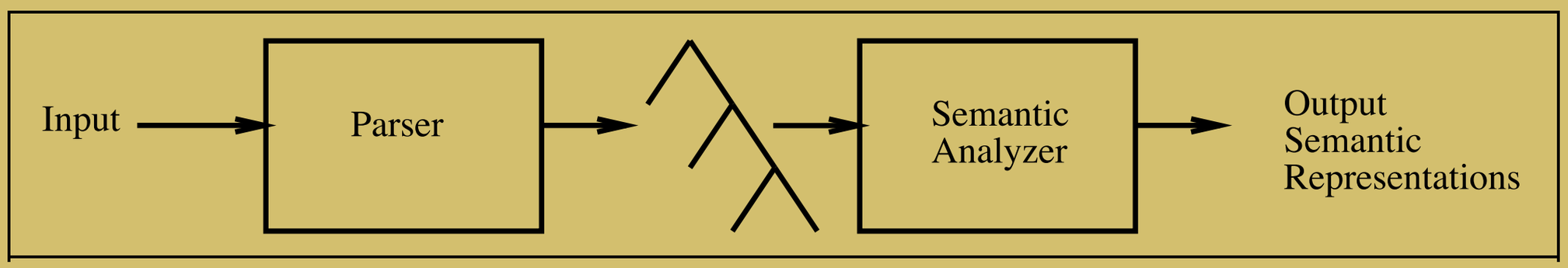
First Order Logic

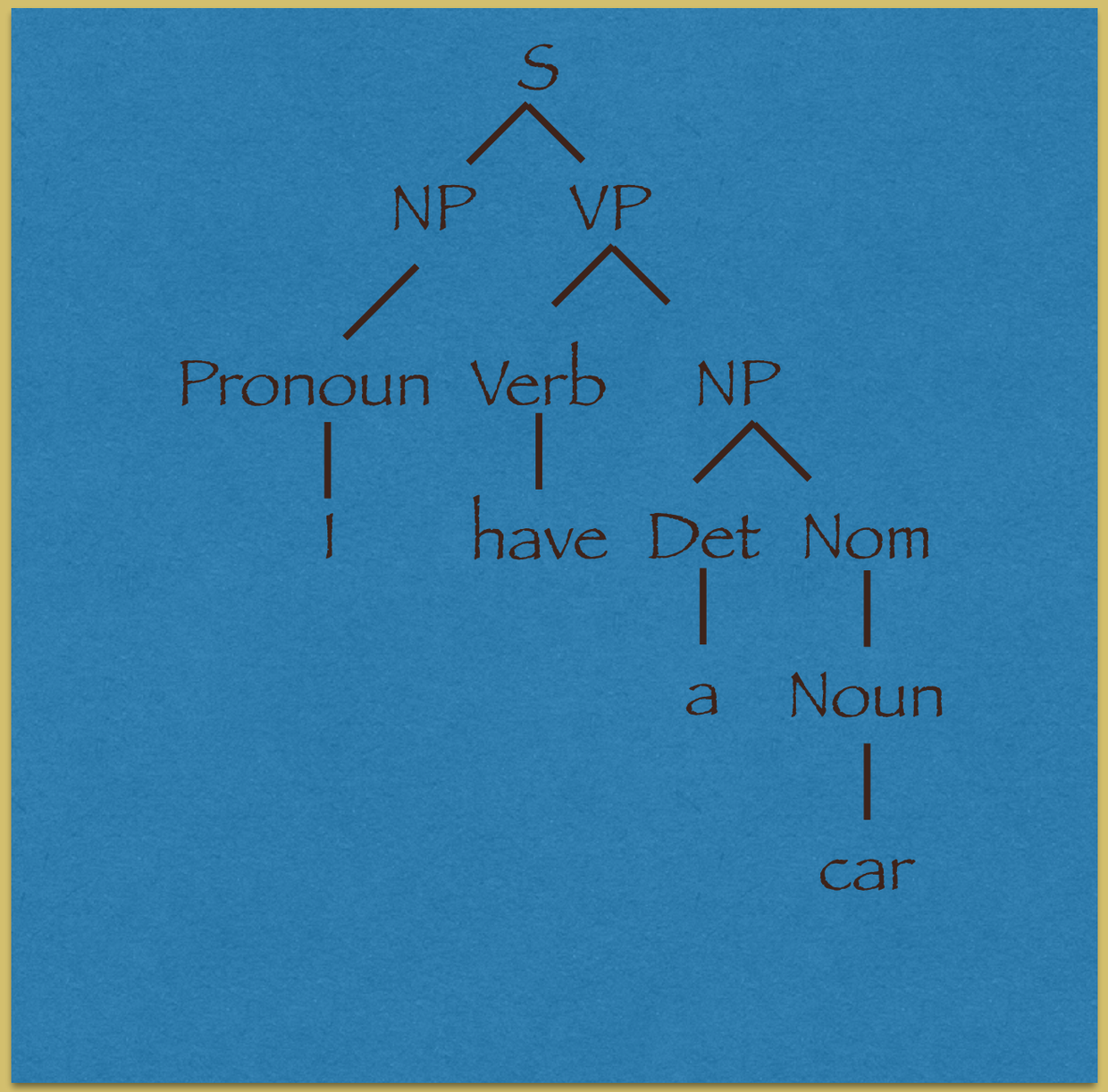
$$\exists x,y Having(x) \wedge Haver(Speaker,x) \wedge HadThing(y,x) \wedge Car(y)$$

The Natural Language Challenge

Rules of Grammar → **Patterns of Meaning**

How to design a formal grammar and a formal semantics, such that the former can be applied to meanings of words to obtain meanings for sentences.





Car

⇑ POSS-BY

Speaker

?

?

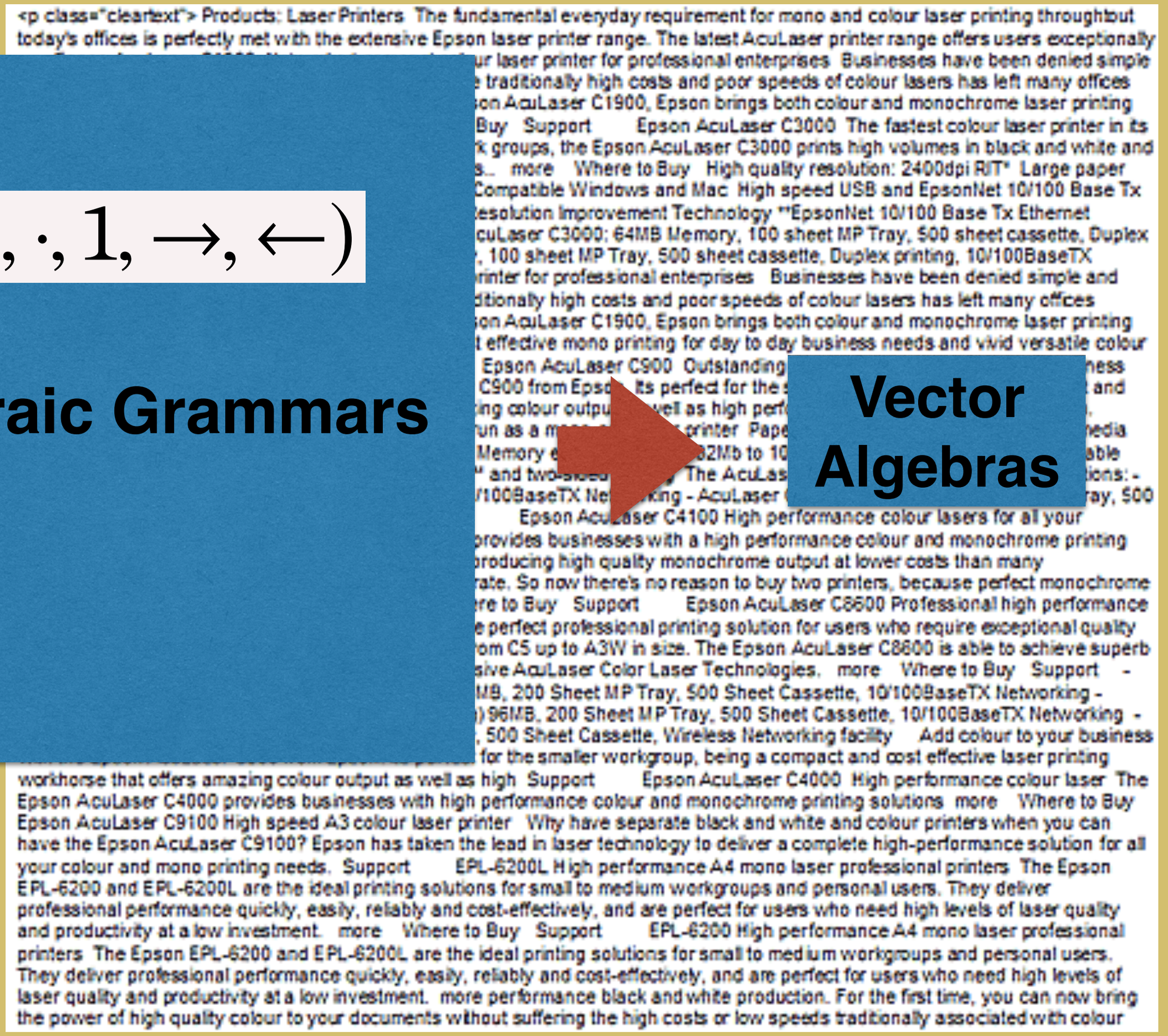
Algebraic Grammars

Vector Algebras

$$(L, \leq, \cdot, 1, \rightarrow, \leftarrow)$$

Algebraic Grammars

Vector Algebras



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AcuLaser C3000: 64MB Memory, 100 sheet MP Tray, 500 sheet cassette, Duplex printing as standard AcuLaser C3000N: 64MB Memory, 100 sheet MP Tray, 500 sheet cassette, Duplex printing, 10/100BaseTX Ethernet Interface Networked compact colour laser printer for professional enterprises Businesses have been denied simple and affordable colour laser printing for far too long. The traditionally high costs and poor speeds of colour lasers has left many offices looking a bit, well, grey. But not any more: with the Epson AcuLaser C1900, Epson brings both colour and monochrome laser printing together at a black and white price. Key Features cost effective mono printing for day to day business needs and vivid versatile colour when required. search Search Epson UK Epson AcuLaser C900 Outstanding professional colour printing for business Add colour to your business with the Epson AcuLaser C900 from Epson. 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Formal Contributions

which grammars ?→ Vector Algebras

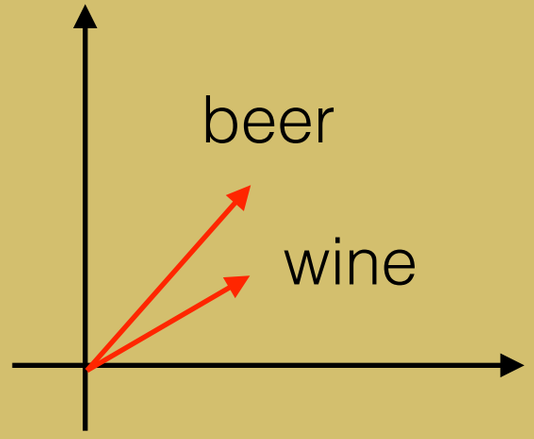
Algebraic Grammars → Tensor Algebras

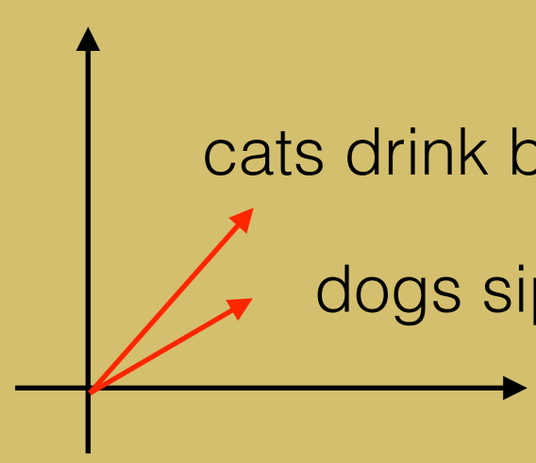
Strongly Monoidal Functor

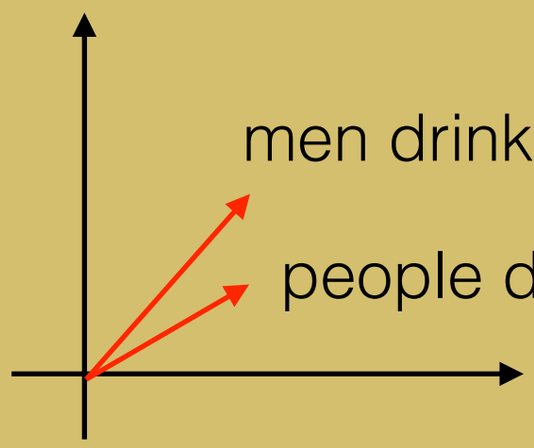
Compact Closed Categories → Compact Closed Categories

Experimental Contributions

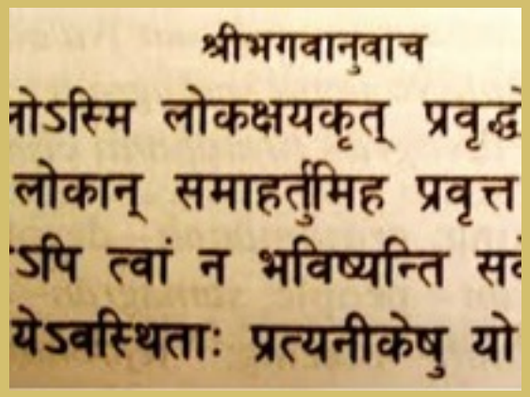
Algebraic Grammars → Tensor Algebras







a bit of History



AG

The first algebraic grammar was written 2000 years ago by Panini for Sanskrit.

Panini used a format resembling our modern day algebraic thinking of Sanskrit expressions.

These rules are still in use.

1935: Ajdukiewicz introduced the first formal algebraic grammar. This has only one rule:

$$B|A \quad A \Rightarrow B$$

expressing a cancelation scheme:

If an expression of grammatical type
A is preceded by an expression of type B|A, we obtain an
expression of type B.

The grammatical type B|A was thought of as

the fraction B over A

The cancelation scheme can be thought of as a multiplication.

B|A A => B

The grammatical type B|A was thought of as

a function from A to B

The cancelation scheme can be thought of as a application.

B|A A => B

Grammaticality

A string of words has a satisfying syntactic connection, iff some ordering of its word types reduce to the distinguished type S via successive uses of the cancelation scheme.

I Prefer a morning flight

I	Prefer	a	morning	flight
NP	(S|NP)|NP	NP|NP	NP|NP	NP

I Prefer a morning flight

NP (S|NP)|NP NP|NP NP|NP NP

NP

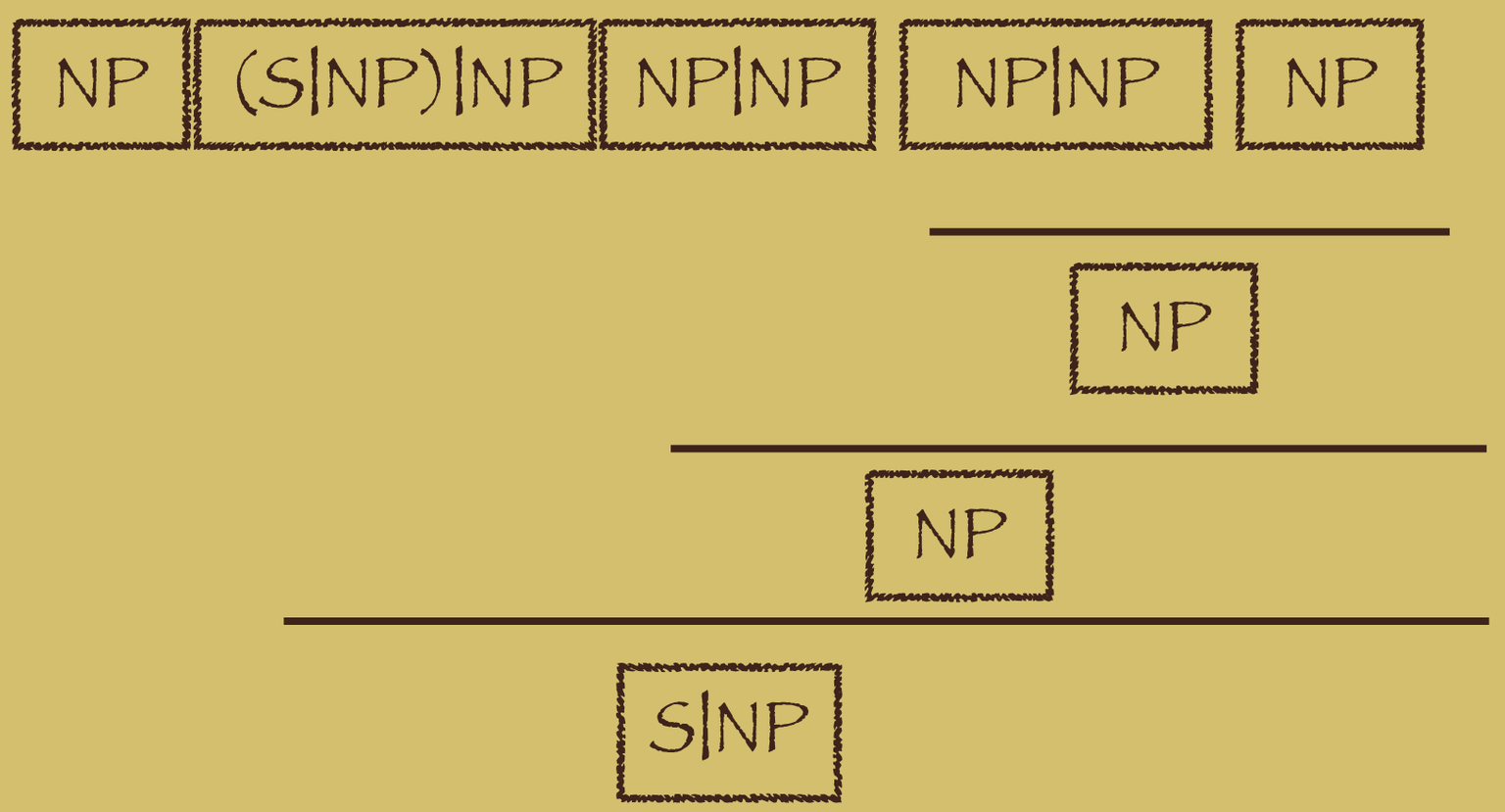
I Prefer a morning flight

NP (S|NP)|NP NP|NP NP|NP NP

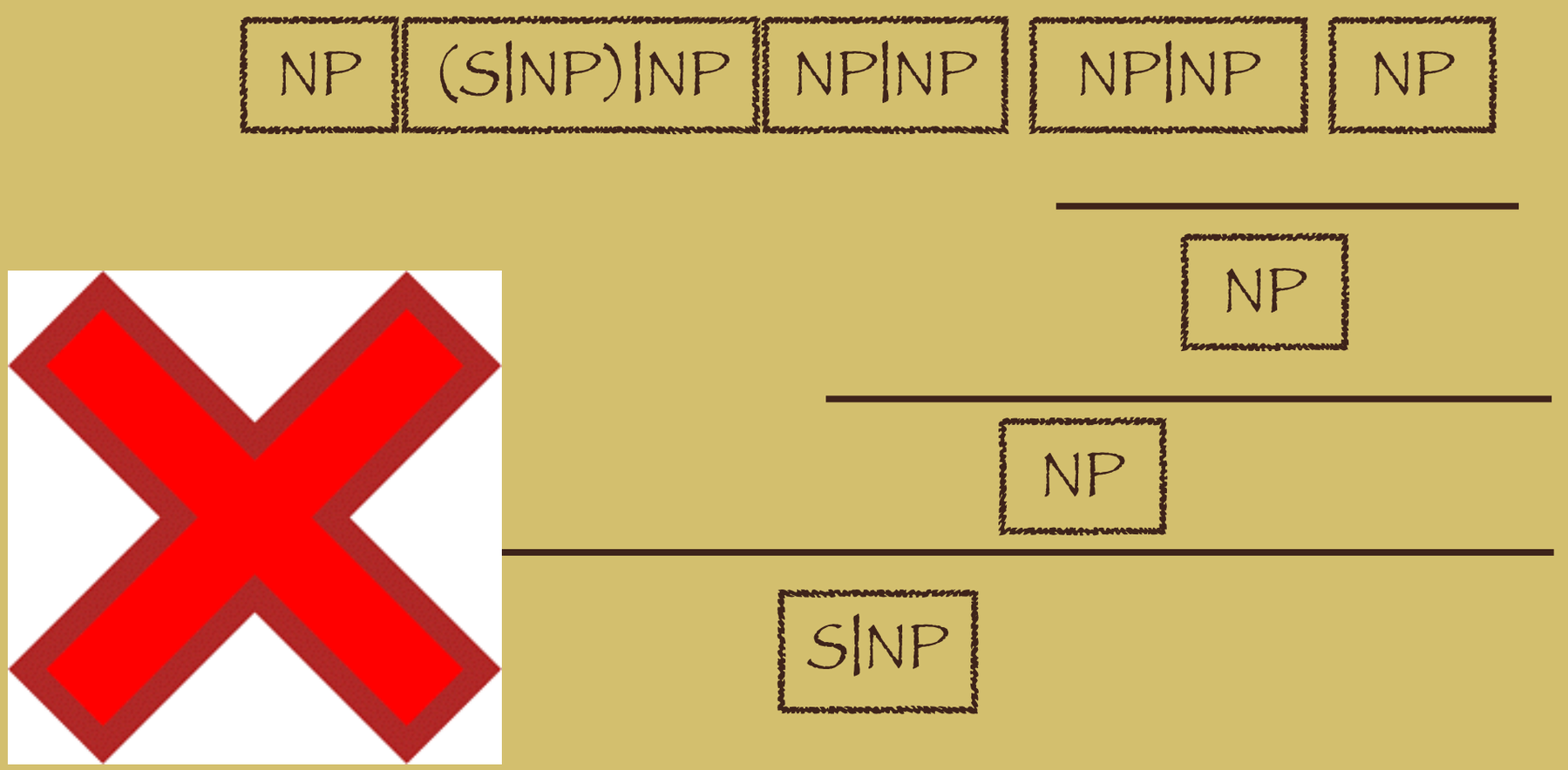
NP

NP

I Prefer a morning flight



I Prefer a morning fight



I Prefer a morning fight

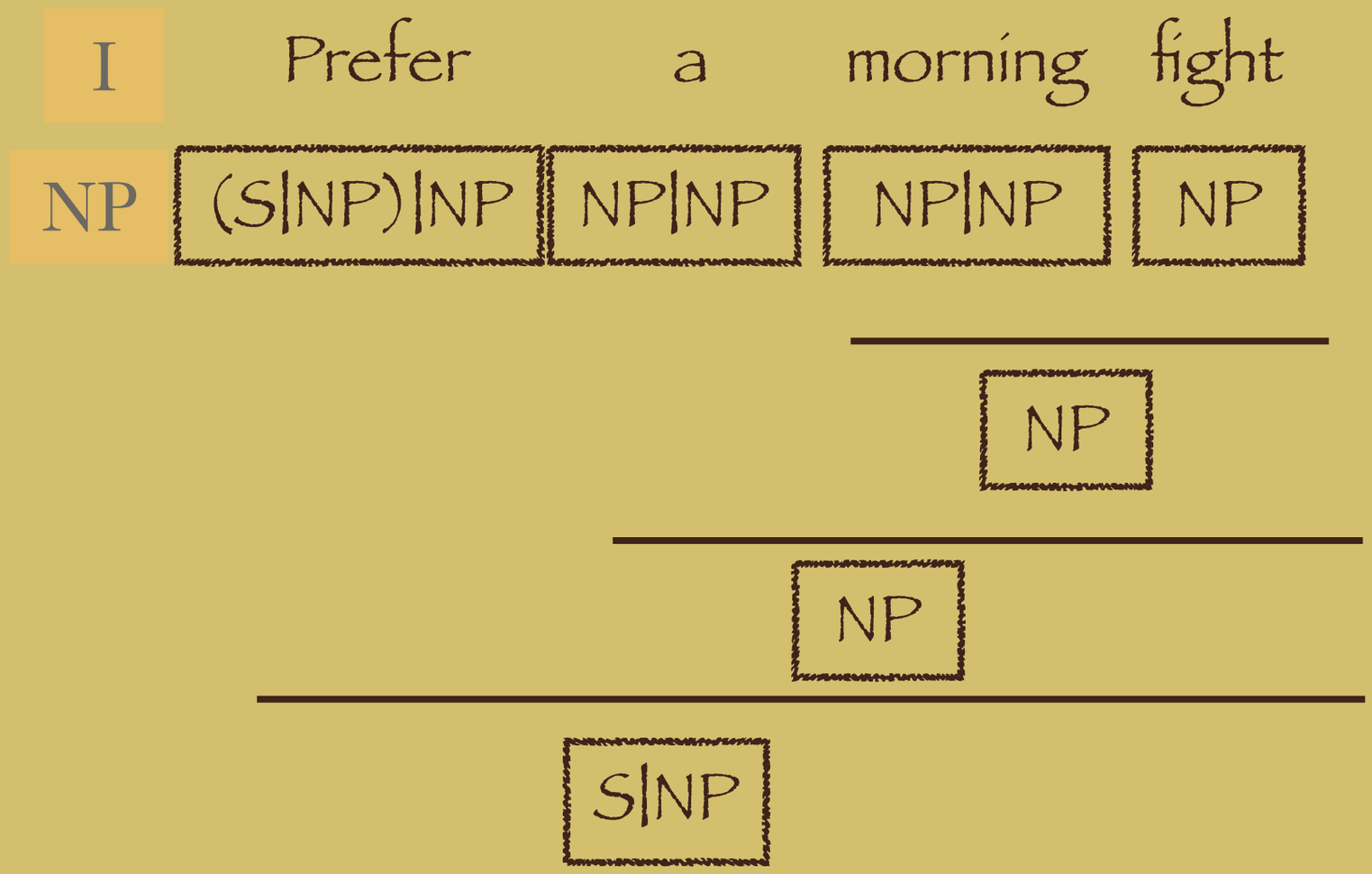
NP (S|NP)|NP NP|NP NP|NP NP

NP

NP

S|NP

Reordering


I
Prefer
a
morning
fight
NP
(S|NP)|NP
NP|NP
NP|NP
NP
NP
NP
S|NP

Prefer a morning fight I

(S|NP)|NP NP|NP NP|NP NP NP

NP

NP

S|NP

S

A refinement.

In 1953 Bar-Hillel introduced directional division types:

A\B and B/A

together with directional cancelation schema:

A A\B => B Backwards

B/A A => B Forwards

The resulting system is called the AB calculus.

I Prefer a morning fight

NP (NP\S)/NP NP/NP NP/NP NP

F → NP

F → NP

F → NP\S

B → S

Prefer a morning fight I

(NP\S)/NP | NP/NP | NP/NP | NP | NP

F

NP

F

NP\S

S

Residuated Monoid

J. Lambek, 1958

Fix the language

Consider an elementary fragment of English consisting of nouns, adjectives, intransitive, transitive verbs.

Consider the rules:

1- An adjective noun phrase is formed by an adjective preceding a noun, as in cute dogs.
2- An intransitive sentence is formed by a noun phrase preceding an intransitive verb, as in men kill.
3- A transitive sentence is formed by a transitive verb occurring between two noun phrases, as in men kill dogs.

Fix a vocabulary

$$\Sigma = \{\text{men}, \text{dogs}, \text{cute}, \text{kill}\}$$

Fix a set of basic types

$$\mathcal{B} = \{n, s\}$$

Define a type assignment

$$\mathcal{D} \subseteq \Sigma \times \mathcal{T}(\mathcal{B})$$

Fix a vocabulary

$$\Sigma = \{\text{men}, \text{dogs}, \text{cute}, \text{kill}\}$$

Fix a set of basic types

$$\mathcal{B} = \{n, s\}$$

An algebra of types

Define a type assignment

$$\mathcal{D} \subseteq \Sigma \times \mathcal{T}(\mathcal{B})$$

Residuated Monoids

$$(L, \cdot, 1)$$

A monoid is a set endowed with a multiplication . that has a unit 1.

A monoid without a unit is a semigroup.

Residuated Monoids

$$(L, \leq, \cdot, 1)$$

A partially ordered a monoid has a partial ordering on it that preserves the multiplication.

$$a \leq c \implies a \cdot b \leq c \cdot b \quad \text{and} \quad b \cdot a \leq b \cdot c$$

Residuated Monoids

$$(L, \leq, \cdot, 1, \rightarrow, \leftarrow)$$

A residuated monoid is a partially ordered a monoid where the multiplication has an adjoint on each of its arguments:

$$\begin{aligned} c \cdot a \leq b \quad &\Longleftrightarrow \quad a \leq c \rightarrow b, \\ a \cdot c \leq b \quad &\Longleftrightarrow \quad a \leq b \leftarrow c. \end{aligned}$$

Residuated Monoids

$$(L, \leq, \cdot, 1, \rightarrow, \leftarrow)$$

Corollaries of these adjunctions are:

$$c \cdot c \rightarrow b \leq b, \quad b \leftarrow c \cdot c \leq b.$$

Residuated Monoids

$$(L, \leq, \cdot, 1, \rightarrow, \leftarrow)$$

Corollaries of these adjunctions are:

$$c \cdot c \rightarrow b \leq b, \quad b \leftarrow c \cdot c \leq b.$$

$$c \wedge c \rightarrow b \quad \leq \quad b$$

Example

An example of a residuated monoid is the set of functions on natural numbers with composition of functions as multiplication and the identity function as its unit.

The ordering is the natural number ordering extended to functions point wisely.

The adjoints are defined using min and max:

$$
\begin{aligned}
c \to b &:= \max\{a \in \mathbb{N} \mid c \cdot a \leq b\}, \\
b \leftarrow c &:= \max\{a \in \mathbb{N} \mid a \cdot c \leq b\}.
\end{aligned}
$$

back to example

$$\Sigma = \{\text{men}, \text{dogs}, \text{cute}, \text{kill}\}$$

$$\mathcal{B} = \{n, s\}$$

$$\mathcal{D} \subseteq \Sigma \times \mathcal{T}(\mathcal{B})$$

The type assignment takes element from the residuated monoid generated over the basic types.

$$\mathcal{D} =$$

$$\{(\text{men}, n), (\text{dogs}, n), (\text{cute}, n \leftarrow n),$$

$$(\text{kill}, n \rightarrow s), (\text{kill}, (n \rightarrow s) \leftarrow n)\}.$$

Example Deductions

men kill

$$n \cdot (n \to s) \leq s\,.$$

men kill dogs

$$\begin{aligned} & n \cdot ((n \to s) \leftarrow n) \cdot n \\ \leq\ & n \cdot (n \to s) \\ \leq\ & s\,. \end{aligned}$$

men kill cute dogs

$$\begin{aligned} & n \cdot ((n \to s) \leftarrow n) \cdot (n \leftarrow n) \cdot n \\ \leq\ & n \cdot ((n \to s) \leftarrow n) \cdot n \\ \leq\ & n \cdot (n \to s) \\ \leq\ & s\,. \end{aligned}$$

… to wrap up

We define a *monoid grammar* to be a tuple

$$(\Sigma, \mathscr{B}, \mathscr{D}, \{s\})$$

where a sequence of words $w_1 w_2 \cdots w_n$ to be a grammatical sentence whenever for $(w_i, t_i) \in \mathscr{D}$ we have:

$$t_1 \cdot t_2 \cdot \cdots \cdot t_n \leq s$$

a grammatical reduction

… to wrap up

We define a *monoid grammar* to be a tuple

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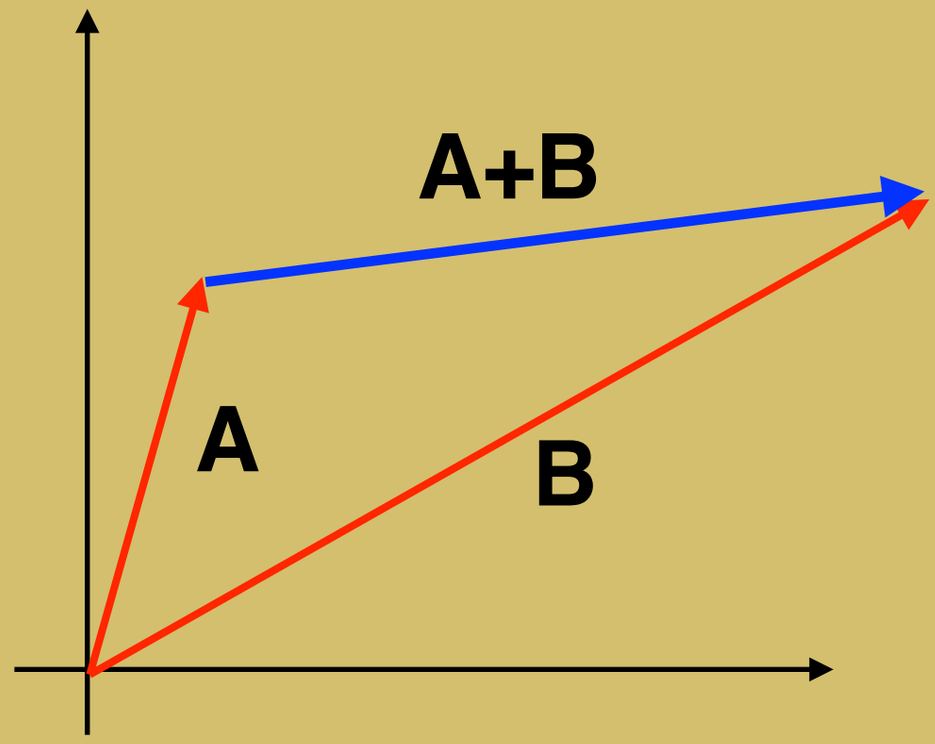
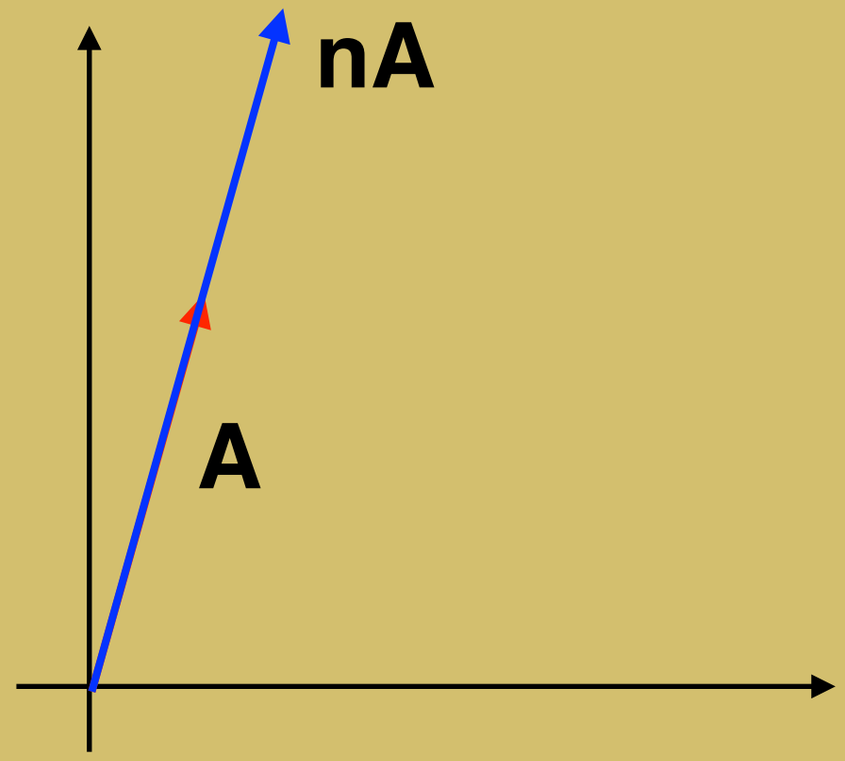
$$t_1 \cdot t_2 \cdot \cdots \cdot t_n \leq s$$

J. Lambek, Mathematics of Syntactic Strucutre, 1958
M. Moortgat, Logical and Linguistic aspects of LC, 1988
G. Morrill, Categorial Grammar: Logical Syntax, Semantics, Processing, 2011

Vector Algebra



Vector Algebra



n(A+B) = nA + nB

Tensor Algebra

1-Dim Tensors

$$T_i \rightsquigarrow \sum_i C_i \overrightarrow{b_i}$$

n-Dim Tensors

$$T_{i_1 i_2 \cdots i_n} \rightsquigarrow \sum_{i_1 i_2 \cdots i_n} C_{i_1 i_2 \cdots i_n} \overrightarrow{b_{i_1}} \otimes \overrightarrow{b_{i_2}} \otimes \cdots \otimes \overrightarrow{b_{i_n}}$$

Tensor Algebra

For $T_{i_1 i_2 \cdots i_n}$ in $\underbrace{W \otimes W \otimes \cdots \otimes W}_{n}$

and $T_{i_n i_{n+1} \cdots i_{n+k}}$ in $\underbrace{W \otimes W \otimes \cdots \otimes W}_{k+1}$

a tensor contraction is $T_{i_1 i_2 \cdots i_{n+1} \cdots i_{n+k}}$

in $\underbrace{W \otimes W \otimes \cdots \otimes W}_{n+k-1}$.

Monoid to Tensor Mapping

$\mathcal{F}$

A mapping between a monoid grammar and tensor products of a finite dimensional vector space W with a fixed set of orthonormal basis is defined as follows:

Monoid to Tensor Mapping

Basic types

$$\mathscr{F}(t) = W \qquad t \in \mathscr{B}.$$

All other types

$$\begin{aligned}
\mathscr{F}(t_1 \cdot t_2) &= \mathscr{F}(t_1) \otimes \mathscr{F}(t_2), \\
\mathscr{F}(t_1 \rightarrow t_2) &= \mathscr{F}(t_1) \otimes \mathscr{F}(t_2), \\
\mathscr{F}(t_1 \leftarrow t_2) &= \mathscr{F}(t_1) \otimes \mathscr{F}(t_2).
\end{aligned}$$

Monoid to Tensor Mapping

Words with atomic type

$$\mathcal{F}(w) = T_i \in W \qquad (w, t) \in \mathcal{D}.$$

All other words

$$\mathcal{F}(w) = T_{i_1 i_2 \cdots i_n} \in \underbrace{W \otimes \cdots \otimes W}_{n}$$

where

$$(w, t_1 \bigcirc \cdots \bigcirc t_n) \in \mathcal{D}$$

.
->, <-

Monoid to Tensor Mapping

The tensor semantics of a string of words

$$w_1 w_2 \cdots w_n$$

is

$$\mathcal{F}(w_1)\mathcal{F}(w_2)\cdots\mathcal{F}(w_n) \ .$$

Examples

men kill $T_{ij}^{\text{kill}} T_j^{\text{men}}$

men kill dogs $(T_{ijk}^{\text{kill}} T_k^{\text{dogs}}) T_j^{\text{men}}$

men kill cute dogs $(T_{ijl}^{\text{kill}} (T_{lk}^{\text{cute}} T_k^{\text{dog}})) T_j^{\text{men}}$

Examples

men kill $T_{ij}^{\text{kill}} T_j^{\text{men}}$

men kill dogs $(T_{ijk}^{\text{kill}} T_k^{\text{dogs}}) T_j^{\text{men}}$

men kill cute dogs $(T_{ijl}^{\text{kill}} (T_{lk}^{\text{cute}} T_k^{\text{dog}})) T_j^{\text{men}}$

Examples

(n->s) <- n

$W \otimes W \otimes W$

men kill $T^{\text{kill}}_{ij} T^{\text{men}}_{j}$

men kill dogs $(T^{\text{kill}}_{ijk} T^{\text{dogs}}_{k}) T^{\text{men}}_{j}$

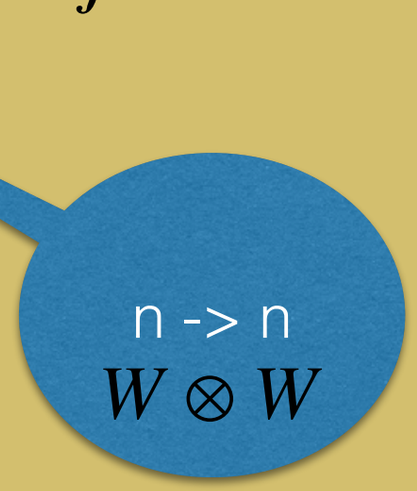
men kill cute dogs $(T^{\text{kill}}_{ijl} (T^{\text{cute}}_{lk} T^{\text{dog}}_{k})) T^{\text{men}}_{j}$

Examples

men kill $T_{ij}^{\text{kill}} T_j^{\text{men}}$

men kill dogs $(T_{ijk}^{\text{kill}} T_k^{\text{dogs}}) T_j^{\text{men}}$

men kill cute dogs $(T_{ijl}^{\text{kill}} (T_{lk}^{\text{cute}} T_k^{\text{dog}})) T_j^{\text{men}}$



… proceed by

Categorical Semantics

Experimental Contributions

Open Problem: from Sentence to Discourse

Monoid Grammer

$\mathcal{F}$

Tensor Algebras

Monoid Grammer

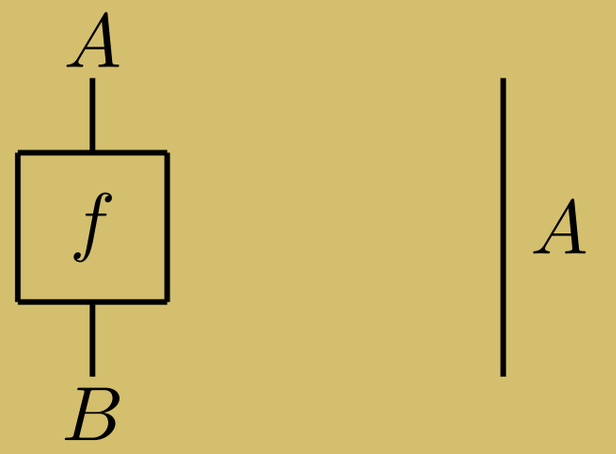
$\mathcal{F}$

?

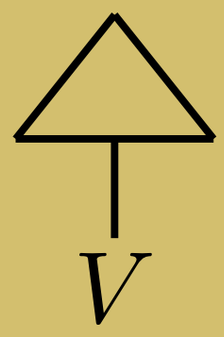
Tensor Algebras

Compact Closed Category

Objects and morphisms



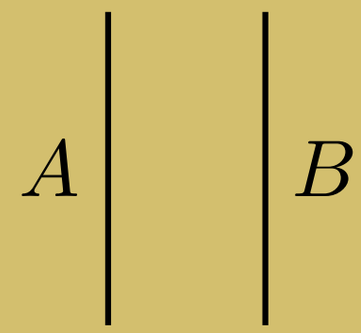
Elements within objects



Compact Closed Category

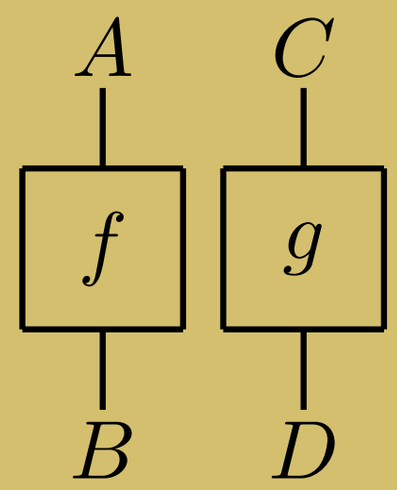
Tensors of objects

$A \otimes B$



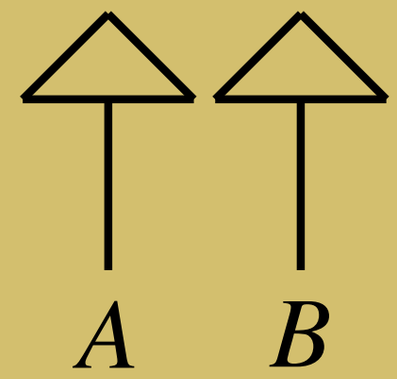
Tensors of morphisms

$f \otimes g$

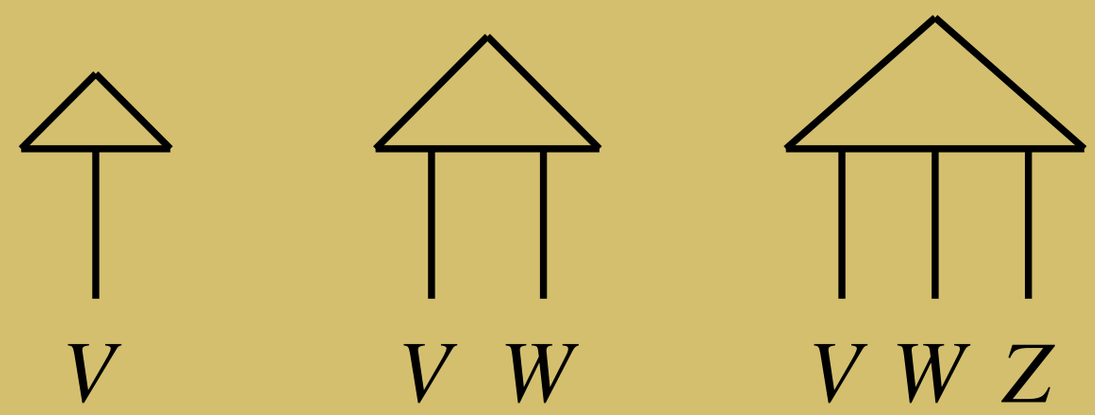


Compact Closed Category

Tensors of elements



Elements of tensors of various ranks

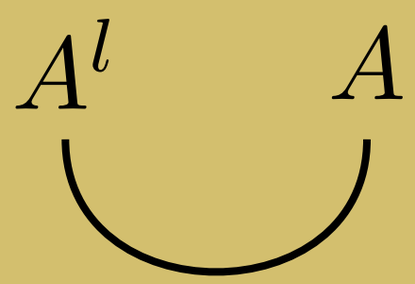


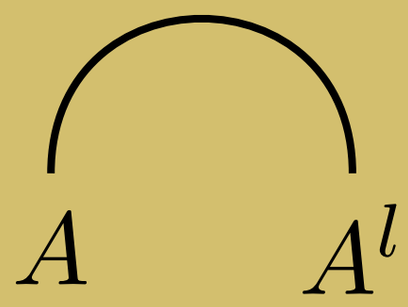
Non Symmetric CCC

Left and right epsilon and eta maps

$$\epsilon^r_A : A \otimes A^r \to I \qquad \eta^r_A : I \to A^r \otimes A$$

$$\epsilon^l_A : A^l \otimes A \to I \qquad \eta^l_A : I \to A \otimes A^l$$



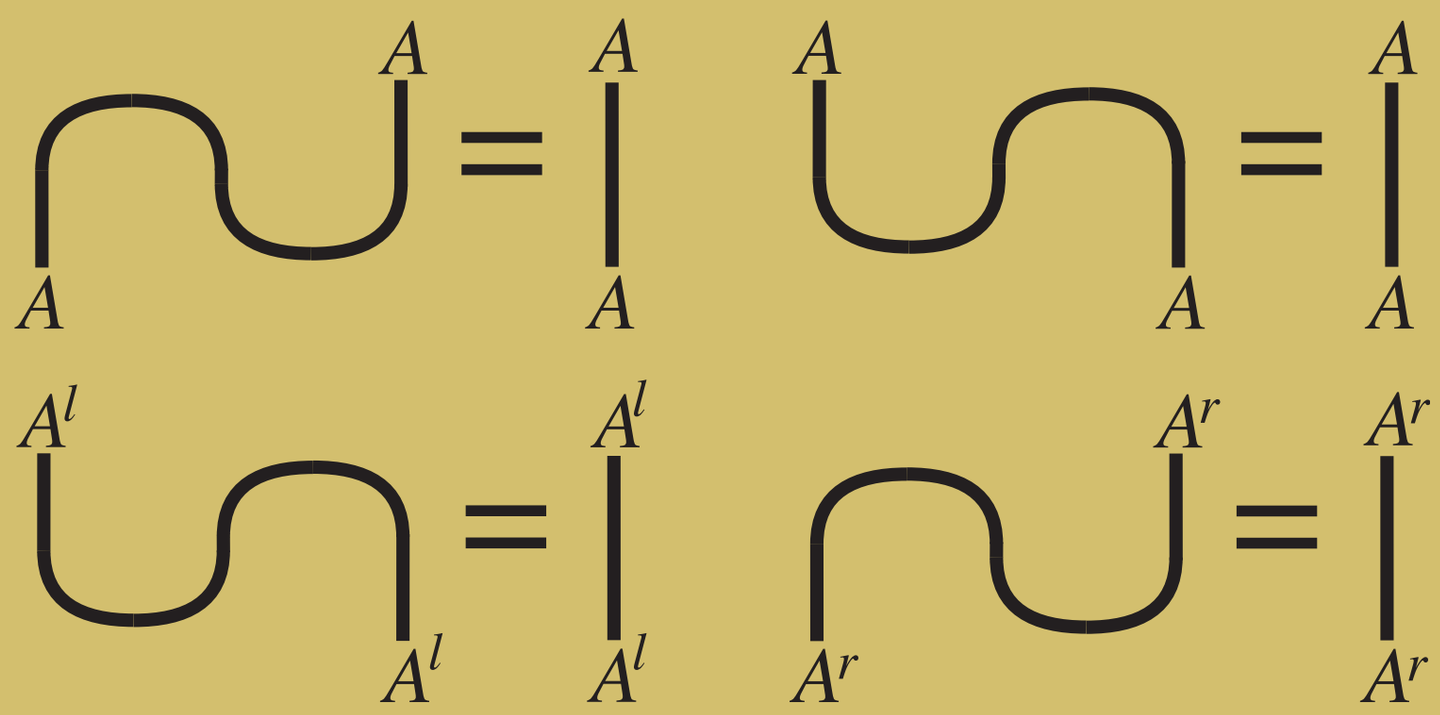


Non Symmetric CCC

satisfying

$$(1_A \otimes \epsilon^l_A) \circ (\eta^l_A \otimes 1_A) = 1_A \qquad (\epsilon^r_A \otimes 1_A) \circ (1_A \otimes \eta^r_A) = 1_A$$

$$(\epsilon^l_A \otimes 1_{A^l}) \circ (1_{A^l} \otimes \eta^l_A) = 1_{A^l} \qquad (1_{A^r} \otimes \epsilon^r_A) \circ (\eta^r_A \otimes 1_{A^r}) = 1_{A^r}$$

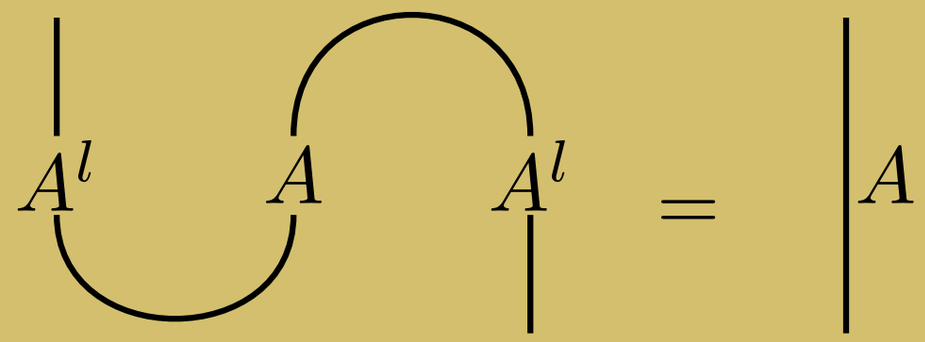


Non Symmetric CCC

For example

$$\epsilon^l : A^l \otimes A \to I, \eta^l : I \to A \otimes A^l$$

Satisfies



Monoidal Functors

A functor between two monoidal categories

$$\mathcal{F} : \mathcal{C} \to \mathcal{D}$$

is monoidal if exists a unit morphism

$$I \to \mathcal{F}(I)$$

and a natural transformation

$$\mathcal{F}(A) \otimes \mathcal{F}(B) \to \mathcal{F}(A \otimes B)$$

A monoidal functor strong if the above have inverses.

These preserve the compact structure.

A strongly monoidal functor on compact closed categories preserves the compact structure.

$$\mathcal{F}(A^l) = \mathcal{F}(A)^l \qquad\qquad \mathcal{F}(A^r) = \mathcal{F}(A)^r$$

To see this, chase the following morphisms:

$$\mathcal{F}(A^l) \otimes \mathcal{F}(A) \to \mathcal{F}(A^l \otimes A) \to \mathcal{F}(I) \to I$$

$$I \to \mathcal{F}(I) \to \mathcal{F}(A \otimes A^l) \to \mathcal{F}(A) \otimes \mathcal{F}(A^l)$$

and recall the uniqueness of adjoints.

… from Residuated Monoids to Pregroups

$$a/b \rightsquigarrow ab^l \qquad\qquad a \setminus b \rightsquigarrow a^r b$$

Pregroup Algebras

Lambek, 1997, LACL

A *pregroup algebra* is a partially ordered monoid, where each element has a left and a right adjoint.

$$P = (P, \leq, \cdot, , 1, (-)^l, (-)^r)$$

$$\forall p \in P, \quad \exists p^l, p^r \in P$$

$$p \cdot p^r \leq 1 \leq p^r \cdot p \qquad p^l \cdot p \leq 1 \leq p \cdot p^l$$

PA's are Compact Closed Categories

elements, adjoints → *objects, adjoints*

partial orders → *arrows*

monoid multiplication, 1 → tensor product, unit

$\epsilon^r_p : p \cdot p^r \to 1$

$\epsilon^l_p : p^l \cdot p \to 1$

contraction

$\eta^r_p : 1 \to p^r \cdot p$

$\eta^l_p : 1 \to p \cdot p^l$

expansion

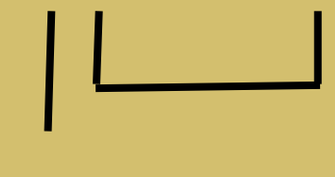
Categorification of Syntax

Preller-Lambek, 2007, MSCS

Grammatical structures become morphisms.

cute dogs

$$nn^l \quad n \xrightarrow{1_n \otimes \epsilon^l_n} n$$



men kill

$$n \quad n^r s \xrightarrow{\epsilon^r_n \otimes 1_s} s$$

men kill dogs

$$n \quad n^r s n^l \quad n \xrightarrow{\epsilon^r_n \otimes 1_s \otimes \epsilon^l_n} s$$

FVect are Compact Closed Categories

vector spaces, adjoints → *objects, dual spaces*

linear maps → *arrows*

tensor product, R → tensor product, unit

Symmetry of tensor

$$\epsilon_V^l = \epsilon_V^r = \epsilon_V : V \otimes V \to \mathbb{R} \qquad \eta_V^l = \eta_V^r = \eta : \mathbb{R} \to V \otimes V$$

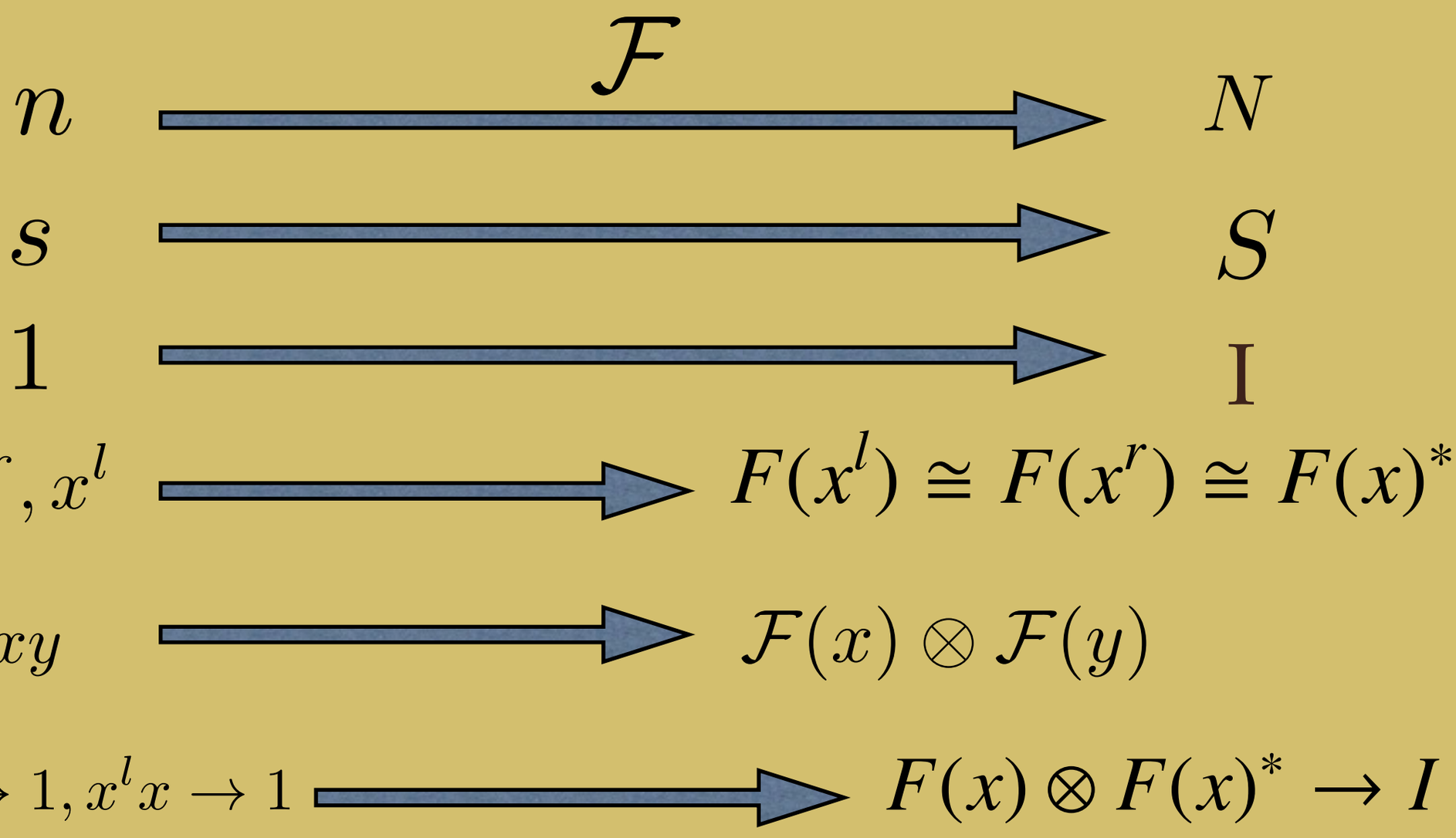
Epsilon map

$$\sum_{ij} C_{ij} \overrightarrow{v}_i \otimes \overrightarrow{v}_j \mapsto \sum_{ij} C_{ij} \langle \overrightarrow{v}_i \mid \overrightarrow{v}_j \rangle$$

Eta map

$$1 \mapsto \sum_i \overrightarrow{v}_i \otimes \overrightarrow{v}_i$$

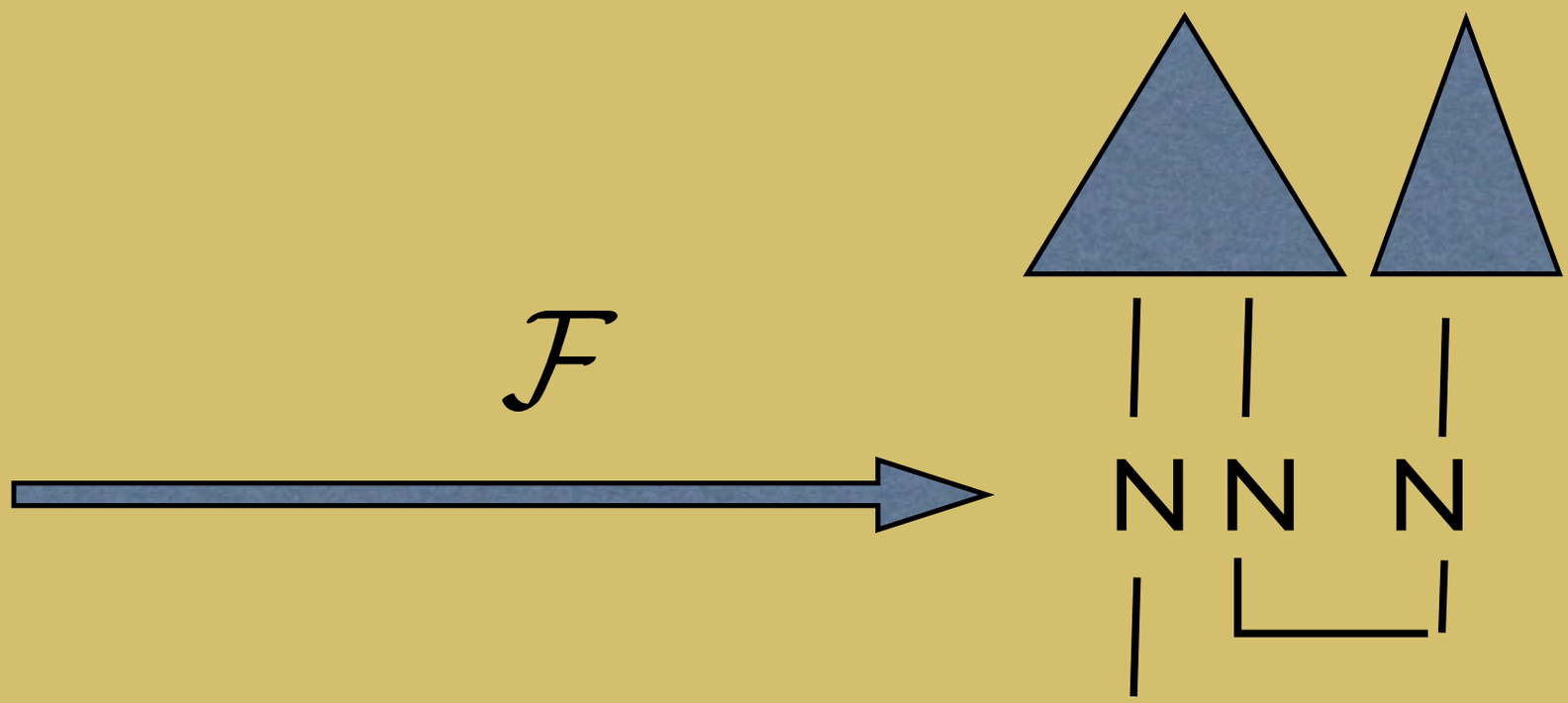
The Passage is a Strongly Monoidal Functor

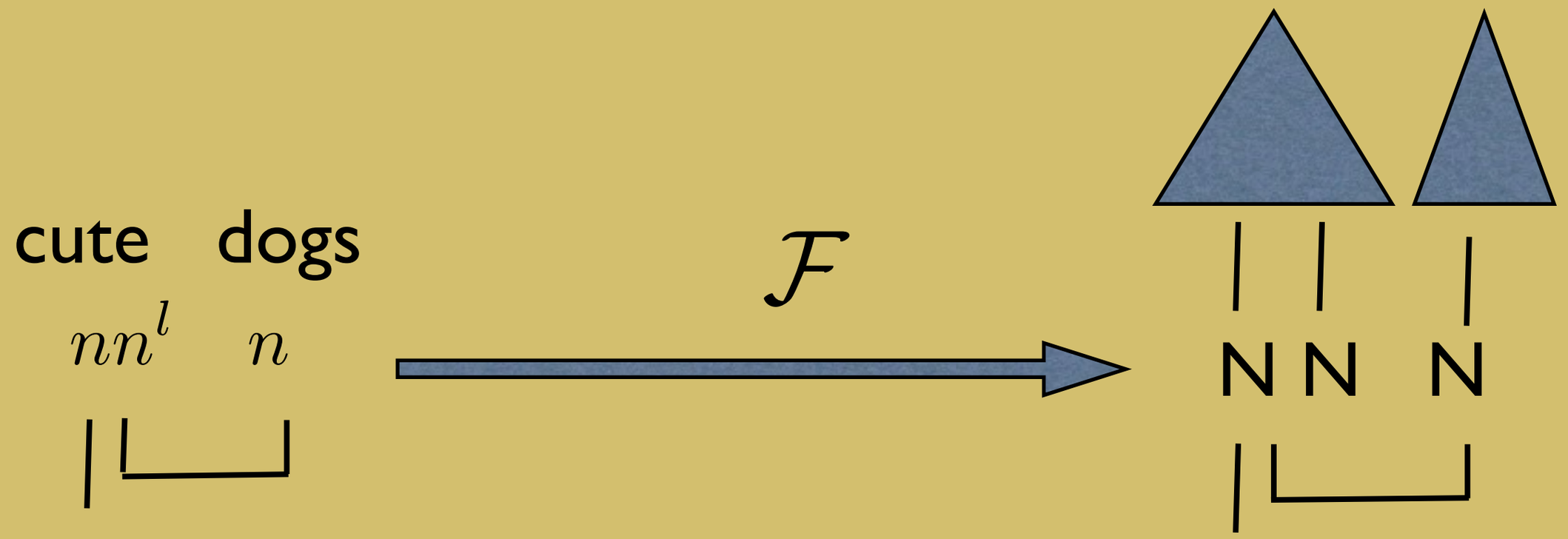


cute dogs

$nn^l \quad n$

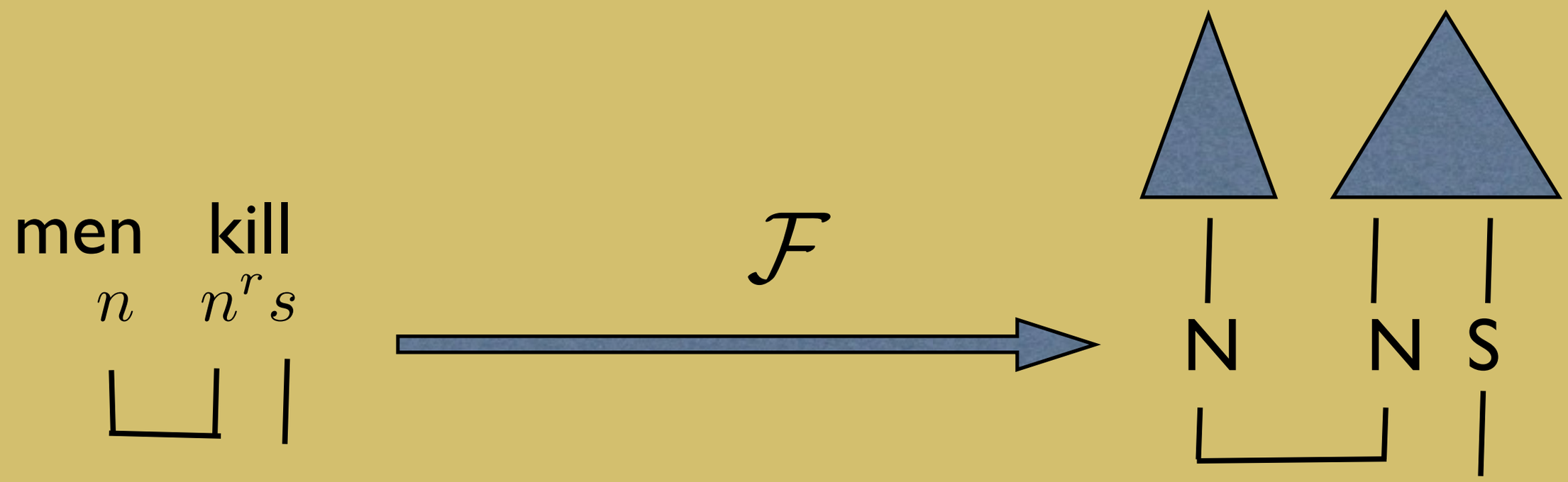
$\mathcal{F}$

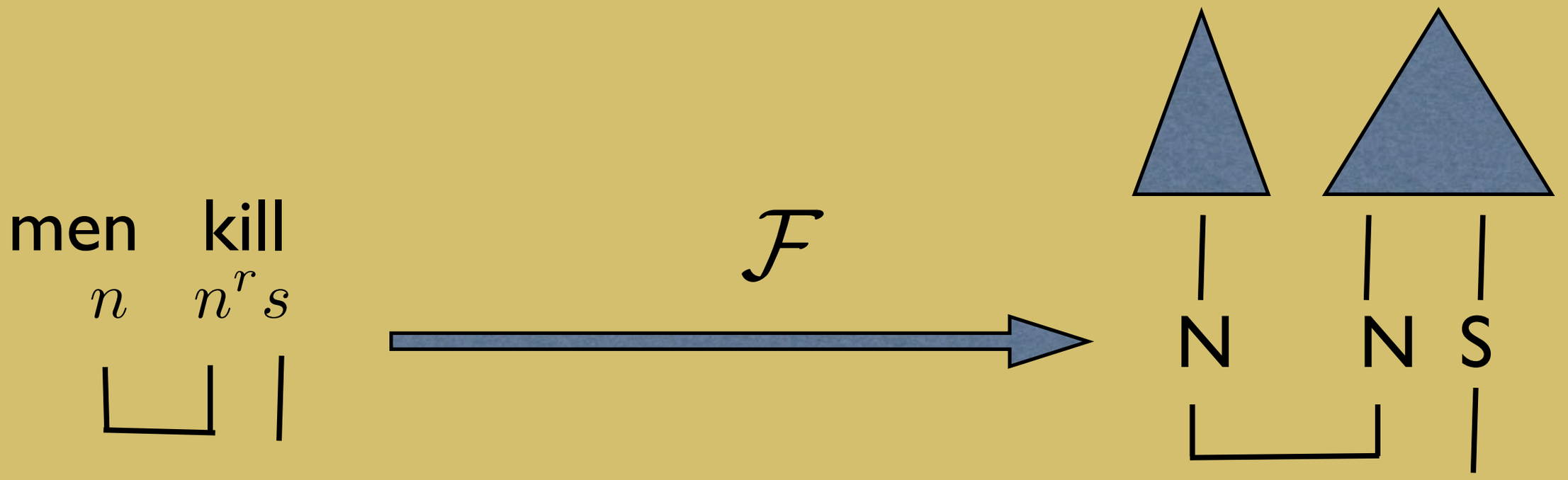




$$\overrightarrow{\text{cute dogs}} = (1_N \otimes \epsilon_N)(\sum_{ij} C_{ij} \overrightarrow{n}_i \otimes \overrightarrow{n}_j \otimes \sum_k C_k \overrightarrow{n}_k)$$

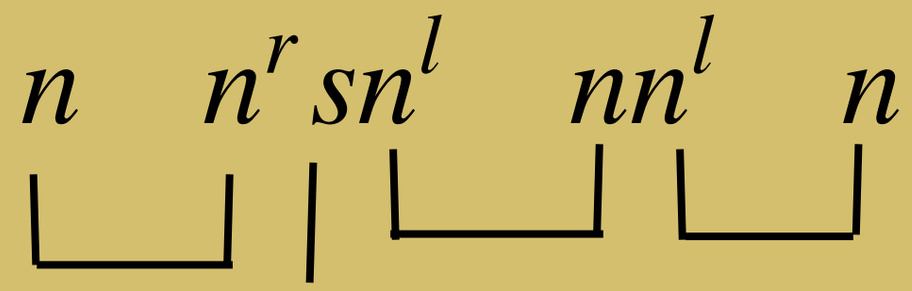
$$= \sum_{ijk} C_{ij} C_k \overrightarrow{n}_i \langle \overrightarrow{n}_j \mid \overrightarrow{n}_k \rangle$$

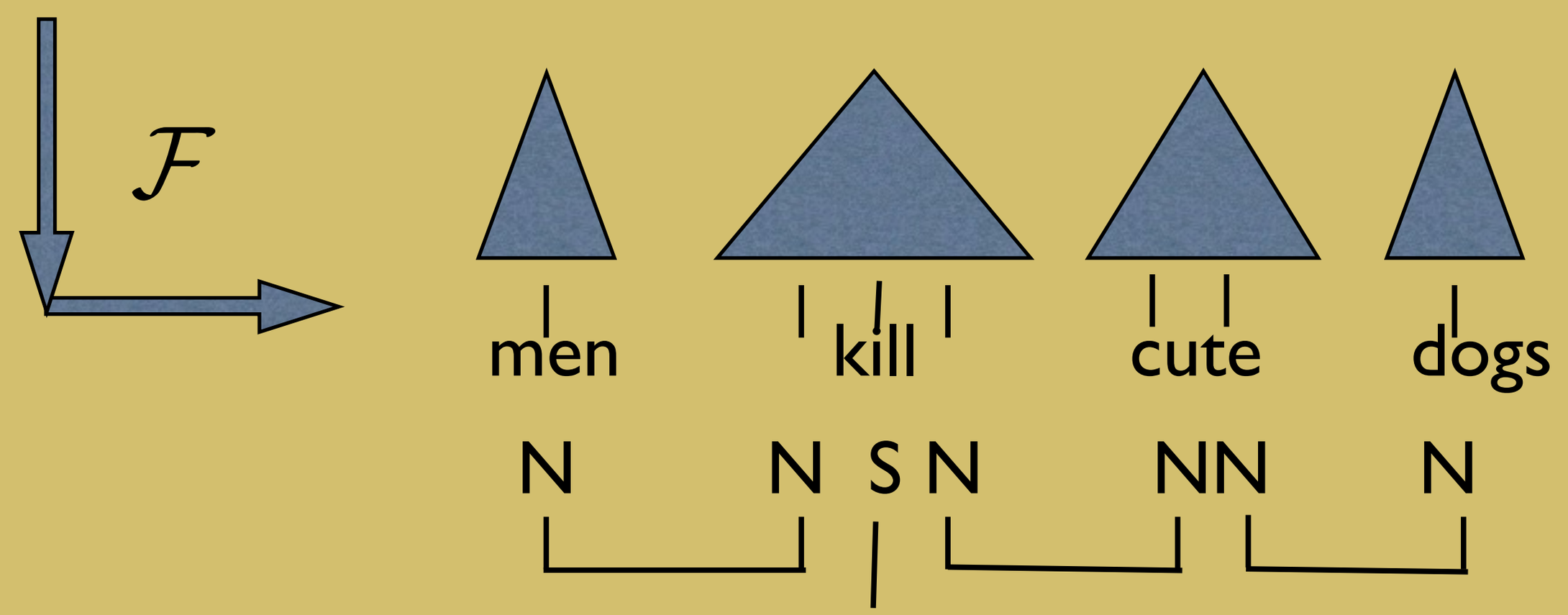

men kill
n n^r s
$\mathcal{F}$
N N S

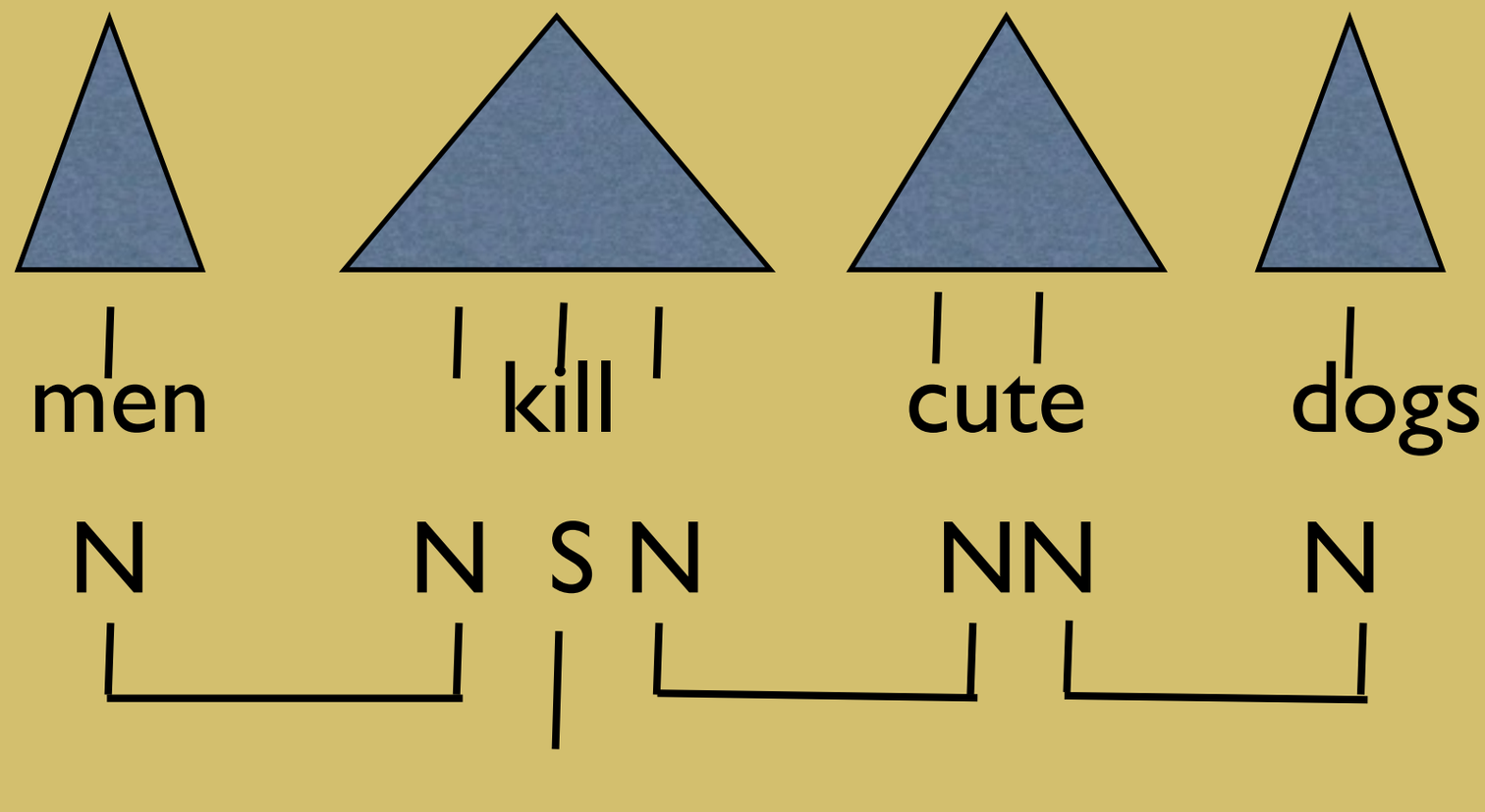


$$\overrightarrow{\text{men kill}} = (\epsilon_N \otimes 1_S)(\sum_i C_i \overrightarrow{n}_i \otimes \sum_{jk} C_{jk} \overrightarrow{n}_j \otimes \overrightarrow{s}_k$$

$$= \sum_{ijk} C_i C_{jk} \langle \overrightarrow{n}_i \mid \overrightarrow{n}_j \rangle \overrightarrow{s}_k$$


n n^r sn^l nn^l n


$\mathcal{F}$
men kill cute dogs
N N S N NN N



$$\overrightarrow{\text{men kill cute dogs}} =$$

$$(\epsilon_N \otimes 1_S \otimes \epsilon_N) \circ (1_{N\otimes N\otimes S\otimes N} \otimes 1_N \otimes \epsilon_N)(\sum_i C_i \overrightarrow{n}_i \otimes \sum_{jkl} C_{jkl} \overrightarrow{n}_j \otimes \overrightarrow{n}_k \otimes \overrightarrow{n}_l \otimes \sum_{mn} C_{mn} \overrightarrow{n}_m \otimes \overrightarrow{n}_n \otimes \sum_o C_o \overrightarrow{n}_o)$$

$$= \sum_{ijklmno} C_i C_{jkl} C_{mn} C_o \langle \overrightarrow{n}_i \mid \overrightarrow{n}_j \rangle \overrightarrow{s}_k \langle \overrightarrow{n}_i \mid \overrightarrow{n}_m \rangle \langle \overrightarrow{n}_n \mid \overrightarrow{n}_o \rangle$$

Pregroup Grammars

Strongly Monoidal Functor

Vector Spaces

Coecke, Sadrzadeh, Clark, Lambek's 90th Festschrift, 2010

Preller, Sadrzadeh, JoLLI, 2011

Coecke, Grefenstette,Sadrzadeh, APAL, 2013

Pregroup Grammars

Strongly Monoidal Functor

Vector Spaces

Can things theory?

NEWSLETTER

Issue: 478 - September 2018

APOLLONIUS CIRCLE COUNTING

GRAPH THEORY AND BOVINE EPIDEMIOLOGY

MATHEMATICS OF LANGUAGE AND GRAMMAR

Krishnamurty and Mitchell, (CVSC, ACL workshop), 2013.

Maillard, Clark, Grefenstette, (Type Theory and NL, EACL workshop), 2014.

Baroni, Bernardini, Zamparelli, (LILT), 2014.

Muskens and Sadrzadeh, DSALT 2016, to apear in JLM 2018

Royal Society International Exchange Award

<p class="cleartext"> Products: Laser Printers The fundamental everyday requirement for mono and colour laser printing throughout today's offices is perfectly met with the extensive Epson laser printer range. The latest AcuLaser printer range offers users exceptionally Epson AcuLaser C1900 Networked compact colour laser printer for professional enterprises Businesses have been denied simple and affordable colour laser printing for far too long. The traditionally high costs and poor speeds of colour lasers has left many offices looking a bit, well, grey. But not any more: with the Epson AcuLaser C1900, Epson brings both colour and monochrome laser printing together at a black and white price. more Where to Buy Support Epson AcuLaser C3000 The fastest colour laser printer in its class The perfect printer for small businesses and work groups, the Epson AcuLaser C3000 prints high volumes in black and white and vibrant colour, at high speed and with low running costs... more Where to Buy High quality resolution: 2400dpi RIT* Large paper capacity: 600 sheets, expandable up to 1,600 sheets Compatible Windows and Mac High speed USB and EpsonNet 10/100 Base Tx Ethernet interfaces as standard** * Epson AcuLaser Resolution Improvement Technology **EpsonNet 10/100 Base Tx Ethernet standard with Epson AcuLaser C3000N model only. AcuLaser C3000: 64MB Memory, 100 sheet MP Tray, 500 sheet cassette, Duplex printing as standard AcuLaser C3000N: 64MB Memory, 100 sheet MP Tray, 500 sheet cassette, Duplex printing, 10/100BaseTX Ethernet Interface Networked compact colour laser printer for professional enterprises Businesses have been denied simple and affordable colour laser printing for far too long. The traditionally high costs and poor speeds of colour lasers has left many offices looking a bit, well, grey. But not any more; with the Epson AcuLaser C1900, Epson brings both colour and monochrome laser printing toget[illegible] a black and white price. Key Features cost effective mono printing for day to day business need[illegible] vivid versatile colour whe[illegible] Epson[illegible] Epson[illegible]900 [illegible] professional colour printing for business Add colour to your business with the Epson AcuLaser C900 from Epson. Its perfect for the smaller workgroup, being a compact and cost effective laser printing workhorse that offers amazing colour output as well as high performance black and white production. more Where to Buy Support As cost efficient to run as a mono-only laser printer Paper capacity of 700 sheets from two media sources Easy to operate with advanced printer driver Memory expandable from 32Mb to 1024Mb Pre-configured models available with Wireless 802.11b, Adobe ® PostScript ® Level 3™ and two sided printing The AcuLaser C1900 is available in 5 configurations: - AcuLaser C1900S: with 32MB, 200 Sheet MP Tray, 10/100 [illegible] - AcuLaser C1900: with 32MB, 200 Sheet MP Tray, 500 Sheet Cassette, 10/100BaseTX Networking Support [illegible]4100 High performance colour lasers for all your business printing needs The Epson AcuLaser C4100 provides businesses with a high performance colour and monochrome printing solution. It adds crucial colour to your business, while producing high quality monochrome output at lower costs than many monochrome-only printers, and it's just as easy to operate. So now there's no reason to buy two printers, because perfect monochrome and colour solutions are available in one. more Where to Buy Support Epson AcuLaser C8600 Professional high performance A3W colour laser printer Epson AcuLaser C8600 is the perfect professional printing solution for users who require exceptional quality colour and mono output on a range of media formats from C5 up to A3W in size. The Epson AcuLaser C8600 is able to achieve superb print quality by utilising a combination of Epson's exclusive AcuLaser Color Laser Technologies. more Where to Buy Support - AcuLaser C1900PS: with Adobe® PostScript® 3™, 96MB, 200 Sheet MP Tray, 500 Sheet Cassette, 10/100BaseTX Networking - AcuLaser C1900D: with Duplex unit (two sided printing) 96MB, 200 Sheet MP Tray, 500 Sheet Cassette, 10/100BaseTX Networking - AcuLaser C1900 WiFi: with 32MB, 200 Sheet MP Tray, 500 Sheet Cassette, Wireless Networking facility Add colour to your business with the Epson AcuLaser C900 from Epson. Its perfect for the smaller workgroup, being a compact and cost effective laser printing workhorse that offers amazing colour output as well as high Support Epson AcuLaser C4000 High performance colour laser The Epson AcuLaser C4000 provides businesses with high performance colour and monochrome printing solutions more Where to Buy Epson AcuLaser C9100 High speed A3 colour laser printer Why have separate black and white and colour printers when you can have the Epson AcuLaser C9100? Epson has taken the lead in laser technology to deliver a complete high-performance solution for all your colour and mono printing needs. Support EPL-6200L High performance A4 mono laser professional printers The Epson EPL-6200 and EPL-6200L are the ideal printing solutions for small to medium workgroups and personal users. 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Google Books: 25 million books and magazines.

Google News: news article from 50,000 sources

Wikipedia: 40 million articles, 301 languages.

British National Corpus: 100 million words

ukWaCkypedia: 200 billion words

Distributional Semantics

"oculist and eye-doctor . . . occur in almost the same environments"

"If A and B have almost identical environments. . . we say that they are synonyms."

Harris (1954)

"You shall know a word by the company it keeps!"

Firth (1957)

Words that occur in similar contexts tend to have similar meanings.

Imagine you had never seen the word marinee, but given a context:

Mareeni is a folkloric creature.
Mainee drinks blood.
Mainee comes alive in dark.
People are scared of *marinee*.

you can guess what it means:

?

Imagine you had never seen the word marinee, but given a context:

Mareeni is a folkloric creature.
Mainee drinks blood.
Mainee comes alive in dark.
People are scared of *marinee*.

you can guess what it means:

something like a vampire

Guess the missing word

It is difficult to make a single, definitive description of the **folkloric** [redacted] though there are several elements common to many European **legends**. [redacted] were usually reported as bloated in appearance, and **ruddy**, **purplish**, or dark in colour; these characteristics were often attributed to the drinking of **blood**. [· · ·] Indeed, **blood** was often seen seeping from the mouth and nose of the [redacted] when it was seen in its **shroud** or **coffin** and its left eye was often open. [· · ·] In Christianity, the [redacted] was viewed as ”a **dead** person who retained a semblance of life and could leave its **grave**-much in the same way that Jesus had risen after his **death** and **burial** and appeared before his followers. In Asia, [· · ·] a [redacted] wanders around animating **dead bodies** at night, attacking the living much like a [redacted]

Solution

It is difficult to make a single, definitive description of the **folkloric** **vampire**, though there are several elements common to many European **legends**. **Vampire** were usually reported as bloated in appearance, and **ruddy**, **purplish**, or dark in colour; these characteristics were often attributed to the drinking of **blood**. [· · ·] Indeed, **blood** was often seen seeping from the mouth and nose of the **vampire** when it was seen in its **shroud** or **coffin** and its left eye was often open. [· · ·] In Christianity, the **vampire** was viewed as ”a **dead** person who retained a semblance of life and could leave its **grave**-much in the same way that Jesus had risen after his **death** and **burial** and appeared before his followers. In Asia, [· · ·] a **vampire** wanders around animating **dead bodies** at night, attacking the living much like a **ghoul**.

Guess the missing word

Butterflie are beautiful, flying insects with large scaly wings. Like all insects, they have six jointed legs, 3 body parts, a pair of antennae, compound eyes, and an exoskeleton. The three body parts are the head, thorax (the chest), and abdomen (the tail end). The butterfly's body is covered by tiny sensory hairs. The four wings and the six legs of the are attached to the thorax. The thorax contains the muscles that make the legs and wings move. Butterflies are very good fliers. They have two pairs of large wings covered with colorful, iridescent scales in overlapping rows. Lepidoptera (butterflies and moths) are the only insects that have scaly wings. The wings are attached to the butterfly's thorax (mid-section). Veins support the delicate wings and nourish them with blood.

Solution

Butterflie are beautiful, flying insects with large scaly wings. Like all insects, they have six jointed legs, 3 body parts, a pair of antennae, compound eyes, and an exoskeleton. The three body parts are the head, thorax (the chest), and abdomen (the tail end). The butterfly's body is covered by tiny sensory hairs. The four wings and the six legs of the butterfly are attached to the thorax. The thorax contains the muscles that make the legs and wings move. Butterflies are very good fliers. They have two pairs of large wings covered with colorful, iridescent scales in overlapping rows. Lepidoptera (butterflies and moths) are the only insects that have scaly wings. The wings are attached to the butterfly's thorax (mid-section). Veins support the delicate wings and nourish them with blood.

Co-Occurence Matrices

sugar, a sliced lemon, a tablespoonful of **apricot** preserve or jam, a pinch each of,
their enjoyment. Cautiously she sampled her first **pineapple** and another fruit whose taste she likened
well suited to programming on the digital **computer**. In finding the optimal R-stage policy from
for the purpose of gathering data and **information** necessary for the study authorized in the

	aardvark	...	computer	data	pinch	result	sugar	...
apricot	0	...	0	0	1	0	1	
pineapple	0	...	0	0	1	0	1	
digital	0	...	2	1	0	1	0	
information	0	...	1	6	0	4	0	

Co-Occurence Matrices

PPMI

	computer	data	pinch	result	sugar
apricot	0	0	2.25	0	2.25
pineapple	0	0	2.25	0	2.25
digital	1.66	0	0	0	0
information	0	0.57	0	0.47	0

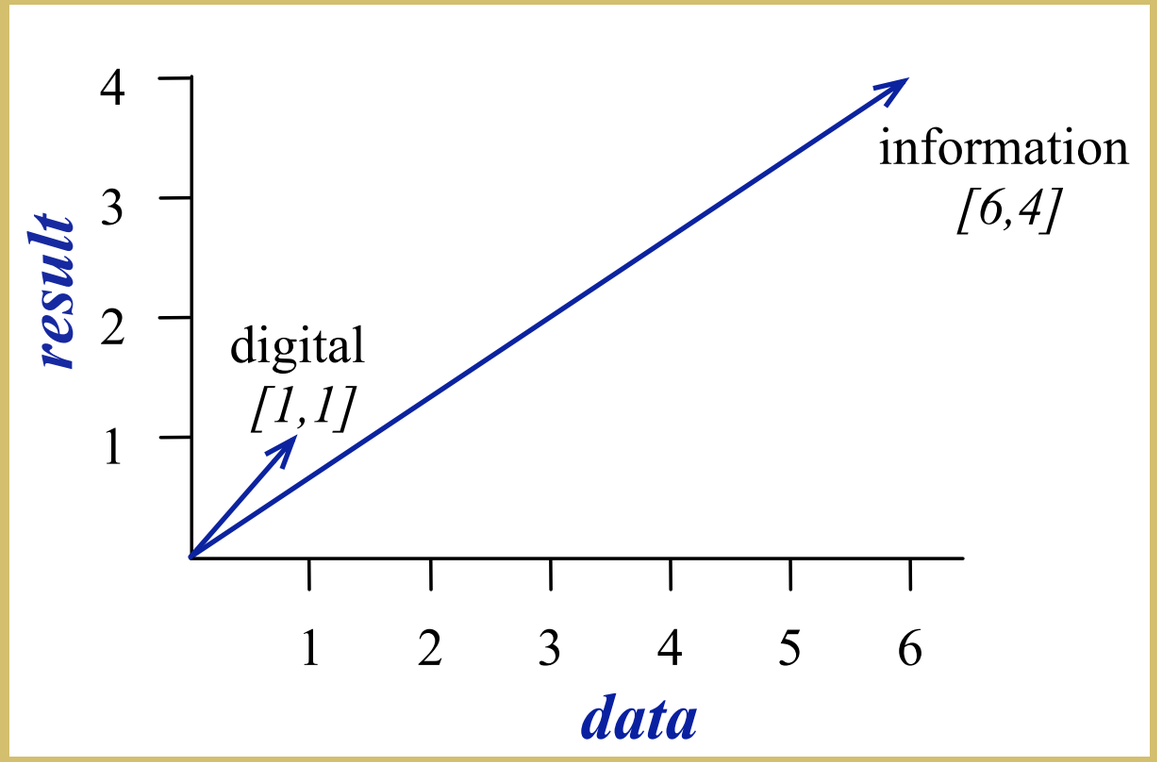
$$\log 2(.05/(.16 * .58)) = -0.618$$

$$\text{PPMI}(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P(c)}, 0)$$

Similarity Measure

PPMI

	computer	data	pinch	result	sugar
apricot	0	0	2.25	0	2.25
pineapple	0	0	2.25	0	2.25
digital	1.66	0	0	0	0
information	0	0.57	0	0.47	0

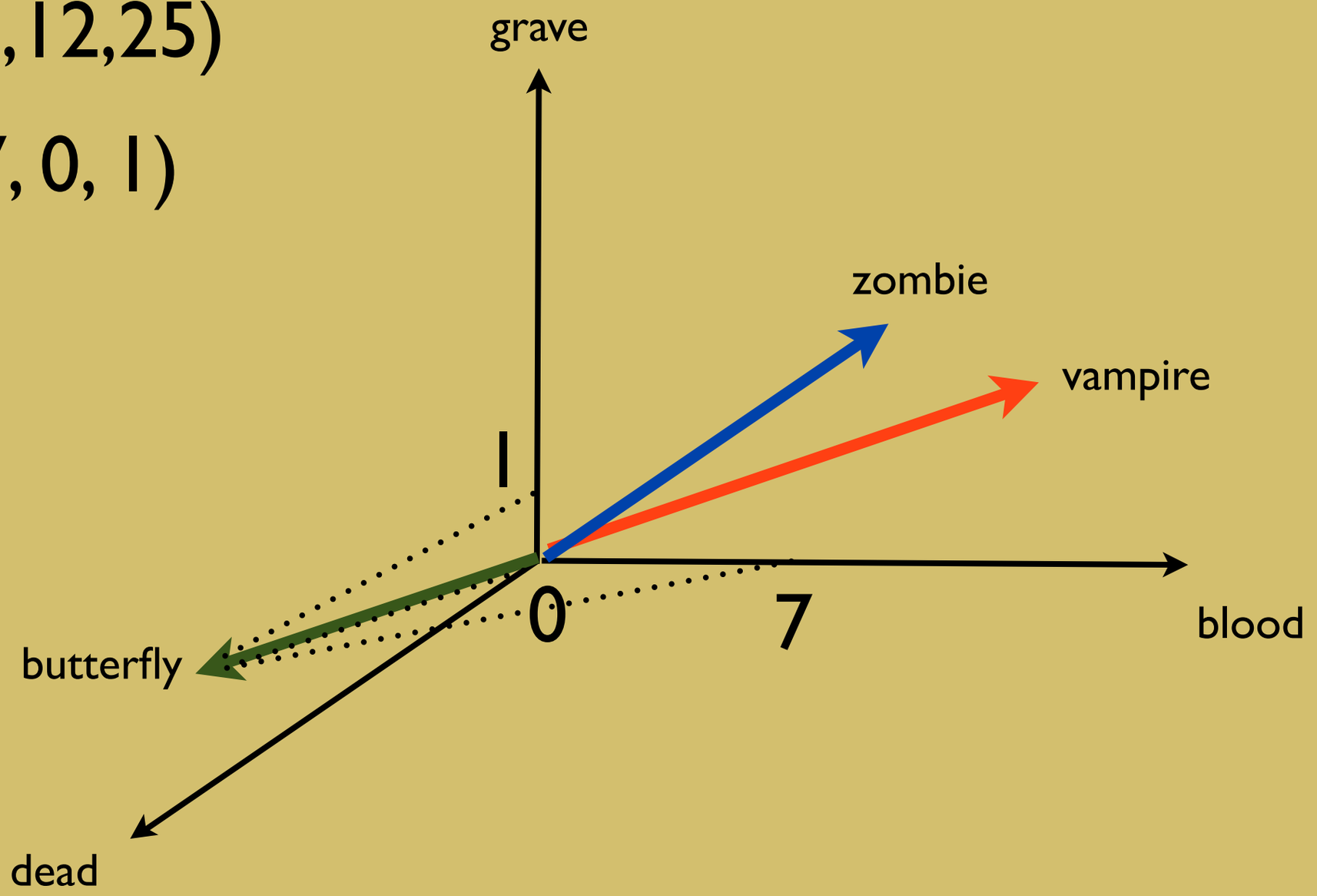


Example

$\overrightarrow{vampire} = (17,13,15)$

$\overrightarrow{zombie} = (16,12,25)$

$\overrightarrow{butterfly} = (7,0,1)$



Evaluations/Applications

WordSim-353: noun pairs

(cup, coffee)

SimLex-999: adjective, noun, and verb pairs

(cup, drink)

TOEFL: 80 questions

"Levied" is closest in meaning to

imposed, believed, requested, correlated

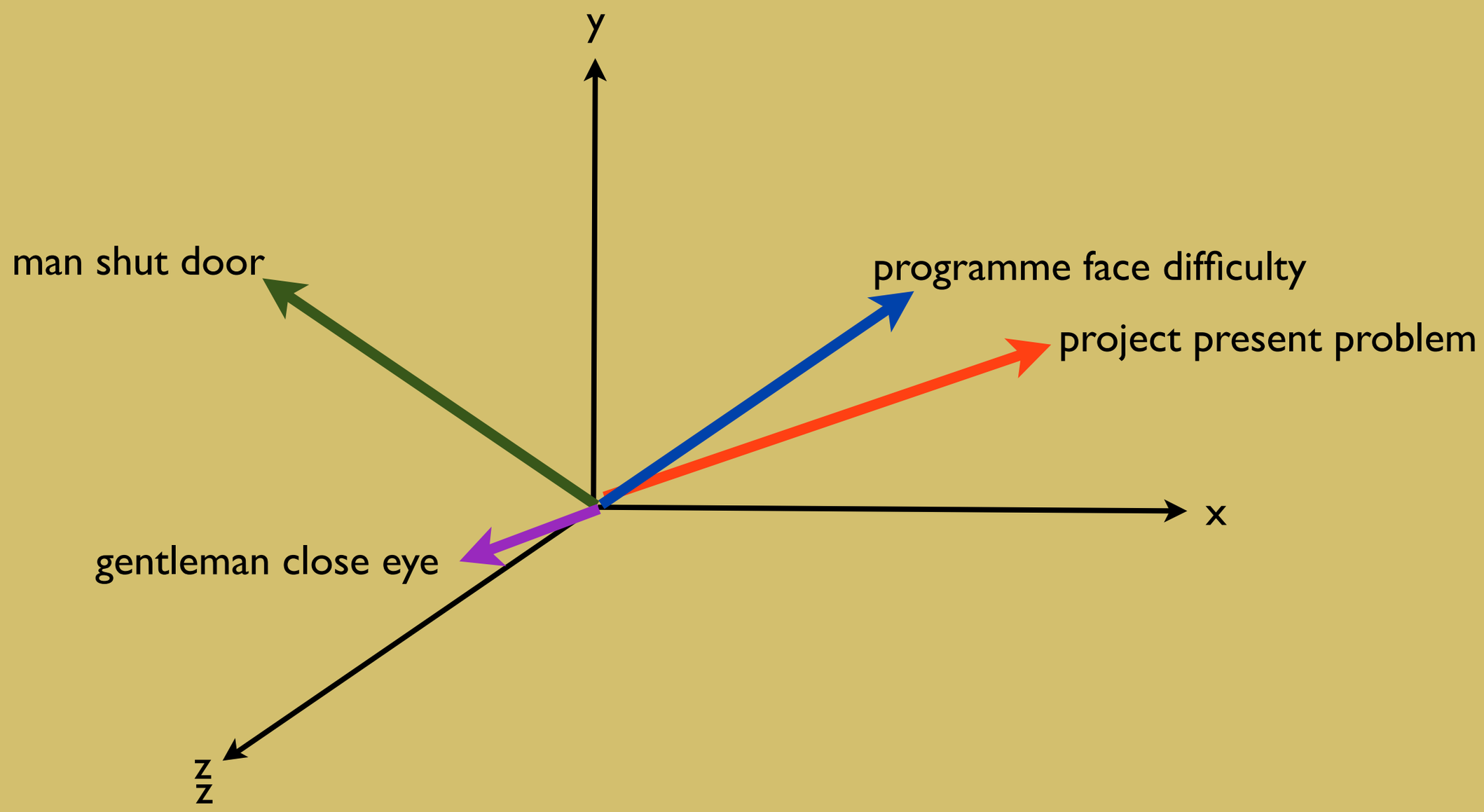
SCWS: 2003 words in sentences

Analogy: a to b is like c to do

Athens to Greece is like Oslo to Norway

At the Sentence Level?

Sentence Similarity

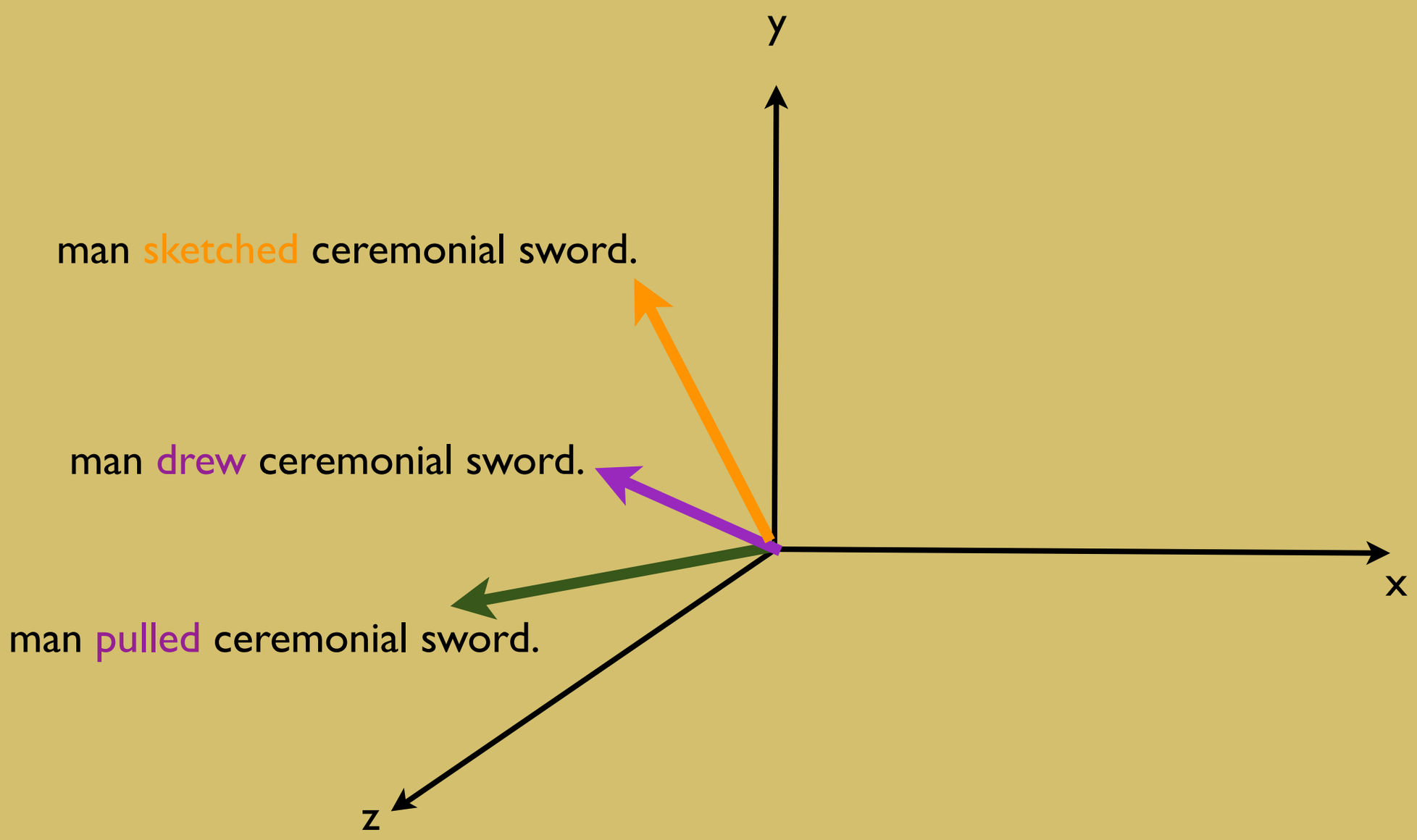


Sentence Similarity

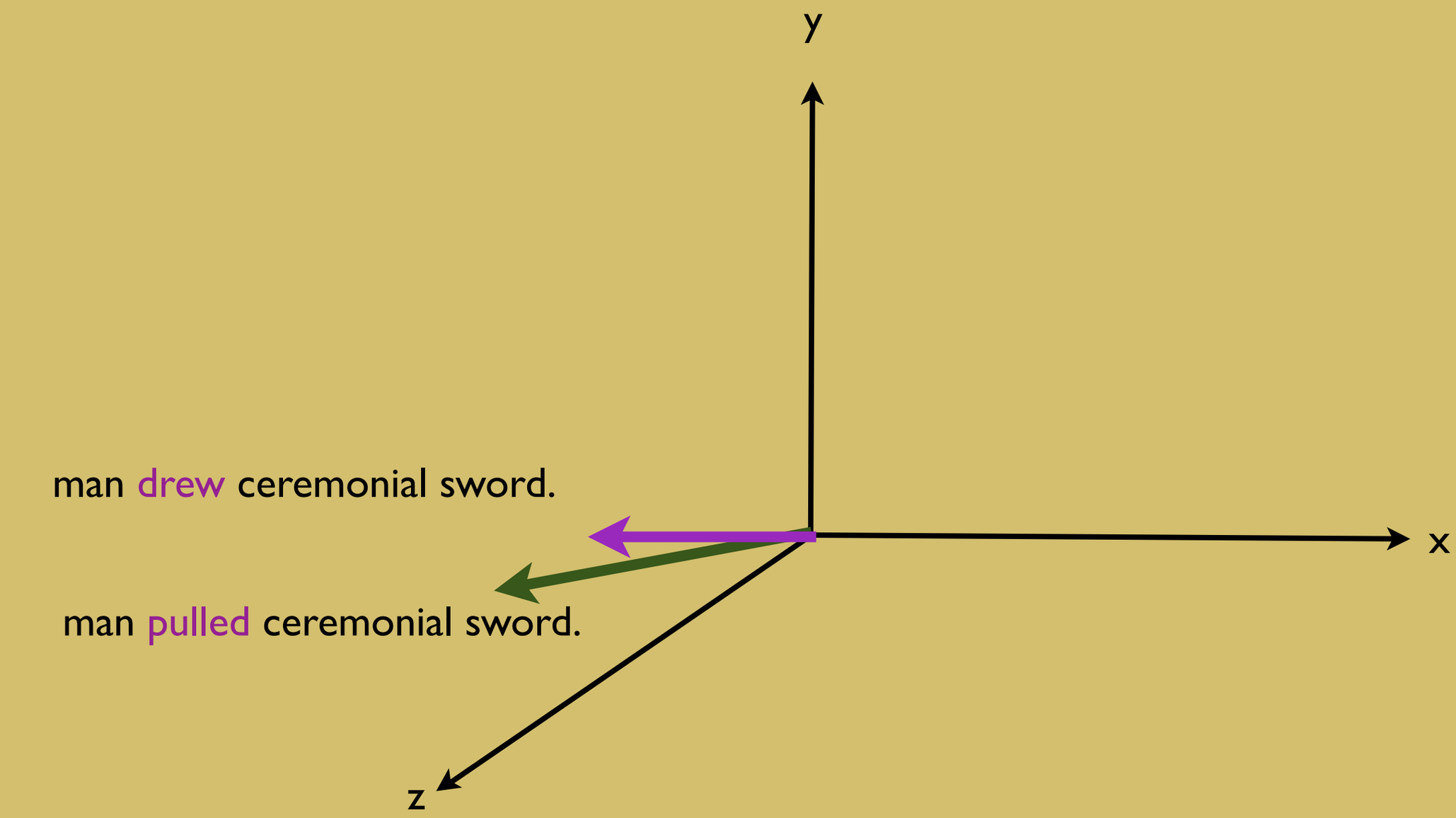
Karsaklis, Sadrzadeh, EMNLP 2013, ACL 2014, CoNLL 2013

	Model	Ambig.		Disamb.
BL	Verbs only	0.310	$\ll$	0.341
M1	Multiplicative	0.325	$\ll$	0.404
M2	Additive	0.368	$\ll$	0.410
T1	Relational	0.368	$\ll$	0.397
T2	**Kronecker**	**0.404**	$<$	**0.412**
T3	Copy-subject	0.310	$\ll$	0.337
$T4_z$	Copy-object	0.321	$\ll$	0.368
	Human agreement	0.550		

Verb Disambiguation



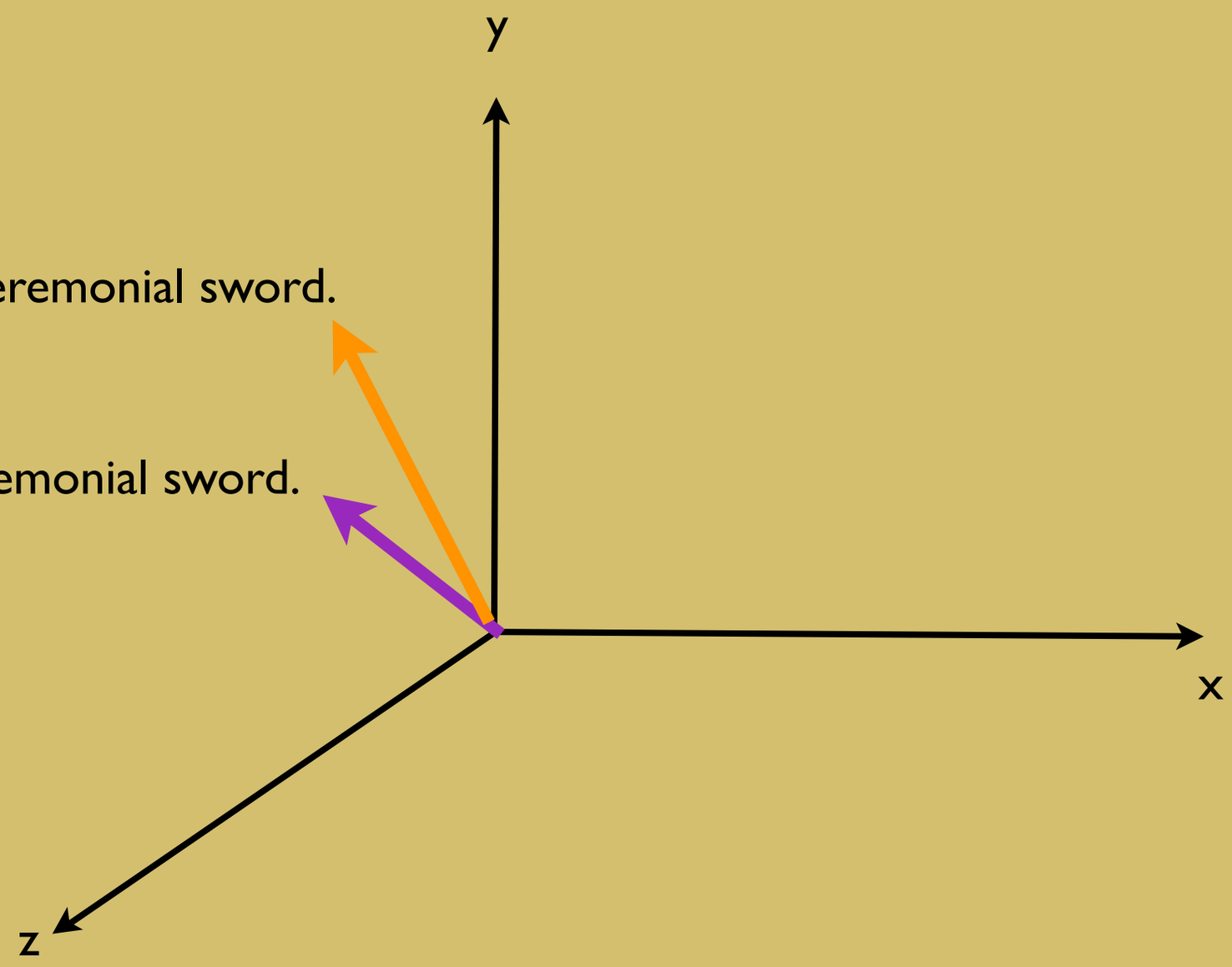
Verb Disambiguation



Verb Disambiguation

man sketched ceremonial sword.

man drew ceremonial sword.



Verb Disambiguation

Grefenstette, Sadrzadeh, J. Comp Ling 2015

Model	ρ
Verb Baseline	0.20
Bigram Baseline	0.14
Trigram Baseline	0.16
Additive	0.10
Multiplicative	
AdjMult	**0.20**
AdjNoun	0.05
CategoricalAdj	**0.20**
Categorical	
AdjMult	0.14
AdjNoun	0.16
CategoricalAdj	**0.19**
Kronecker	
AdjMult	**0.26**
AdjNoun	0.17
CategoricalAdj	**0.27**
Upperbound	0.48

Verb Disambiguation

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Verb Disambiguation

Milajevs, Kartsaklis, Sadrzadeh, Purver EMNLP 2014

Method	**GS11**	**KS14**	**NWE**
Verb only	0.212	0.325	0.107
Addition	0.103	0.275	0.149
Multiplication	0.348	0.041	0.095
Kronecker	0.304	0.176	0.117
Relational	0.285	0.341	0.362
Copy subject	0.089	0.317	0.131
Copy object	0.334	0.331	**0.456**
Frobenius add.	0.261	0.344	0.359
Frobenius mult.	0.233	0.341	0.239
Frobenius outer	0.284	0.350	0.375

Sentence Entailment

Subject-verb-object
report describe result ⊢ document explain process
report outline progress ⊢ document describe change
value suit budget ⊢ number meet standard
book present account ⊢ work show evidence
woman marry man ⊢ female join male
author retain house ⊢ person hold property

Sentence Entailment

Balkir, Kartsaklis, Sdrzadeh, ISAIM, LACL, COLING 2016

Annals of Maths and AI, 2018

Model	Inclusion	KL-div	αSkew	WeedsPrec	ClarkeDE	APinc	balAPinc	SAPinc	SBalAPinc
Verb	0.61	0.61	0.66	0.69	0.58	0.74	0.67	0.59	0.63
$\odot$	0.55	0.65	0.74	0.79	0.67	0.76	0.71	**0.80**	**0.80**
min	0.55	0.71	0.74	0.78	0.63	0.77	0.71	0.73	0.76
$+$	0.58	0.54	0.71	0.59	0.60	0.65	0.64	0.67	0.67
max	0.58	0.55	0.68	0.58	0.58	0.63	0.61	0.60	0.61
Least-Sqr	–	–	–	–	–	–	–	–	–
$\otimes_{\text{rel}}$	0.51	0.64	0.78	0.79	0.69	0.79	0.72	0.84	0.83
$\otimes_{\text{proj}}$	0.64	0.60	0.70	0.69	0.61	0.74	0.70	0.75	0.76
$\otimes_{\text{CpSbj}}$	0.57	0.65	0.73	0.77	0.63	0.73	0.68	0.79	0.78
$\otimes_{\text{CpObj}}$	0.54	0.62	0.73	0.72	0.64	0.76	0.71	0.81	0.79
$\otimes_{\text{FrAdd}}$	0.60	0.60	0.75	0.72	0.67	0.77	0.75	0.84	0.82
$\otimes_{\text{FrMul}}$	0.55	0.62	0.76	0.81	0.68	0.78	0.73	**0.86**	**0.83**

At the Discourse Level?

Anaphora and Ellipsis

Cats chase dogs, children too.

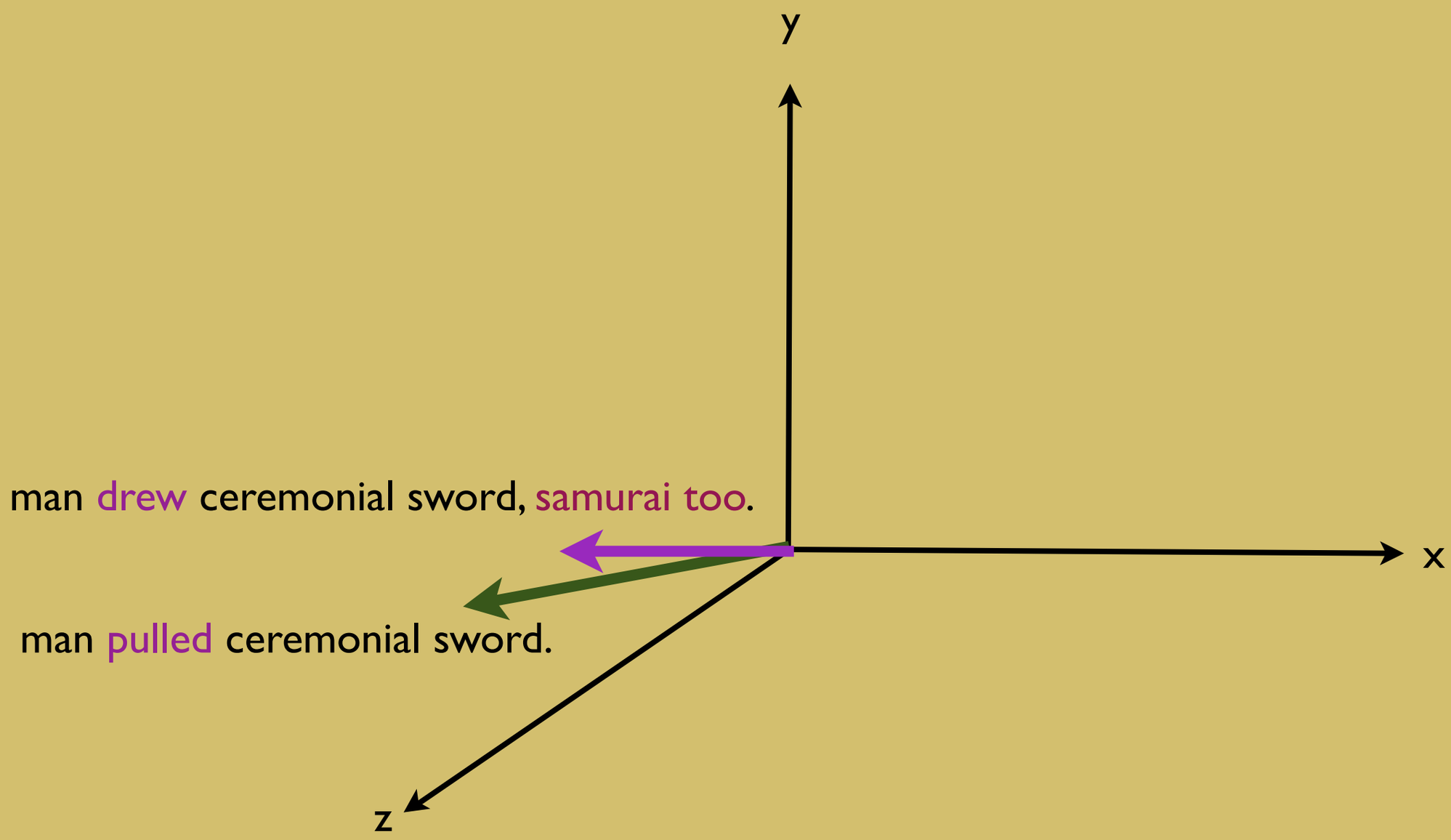
Cats chase dogs. Children do too.

Cats chase their tales, dogs too.

Cats chase themselves, dogs too.

Cats miaow. They are cute.

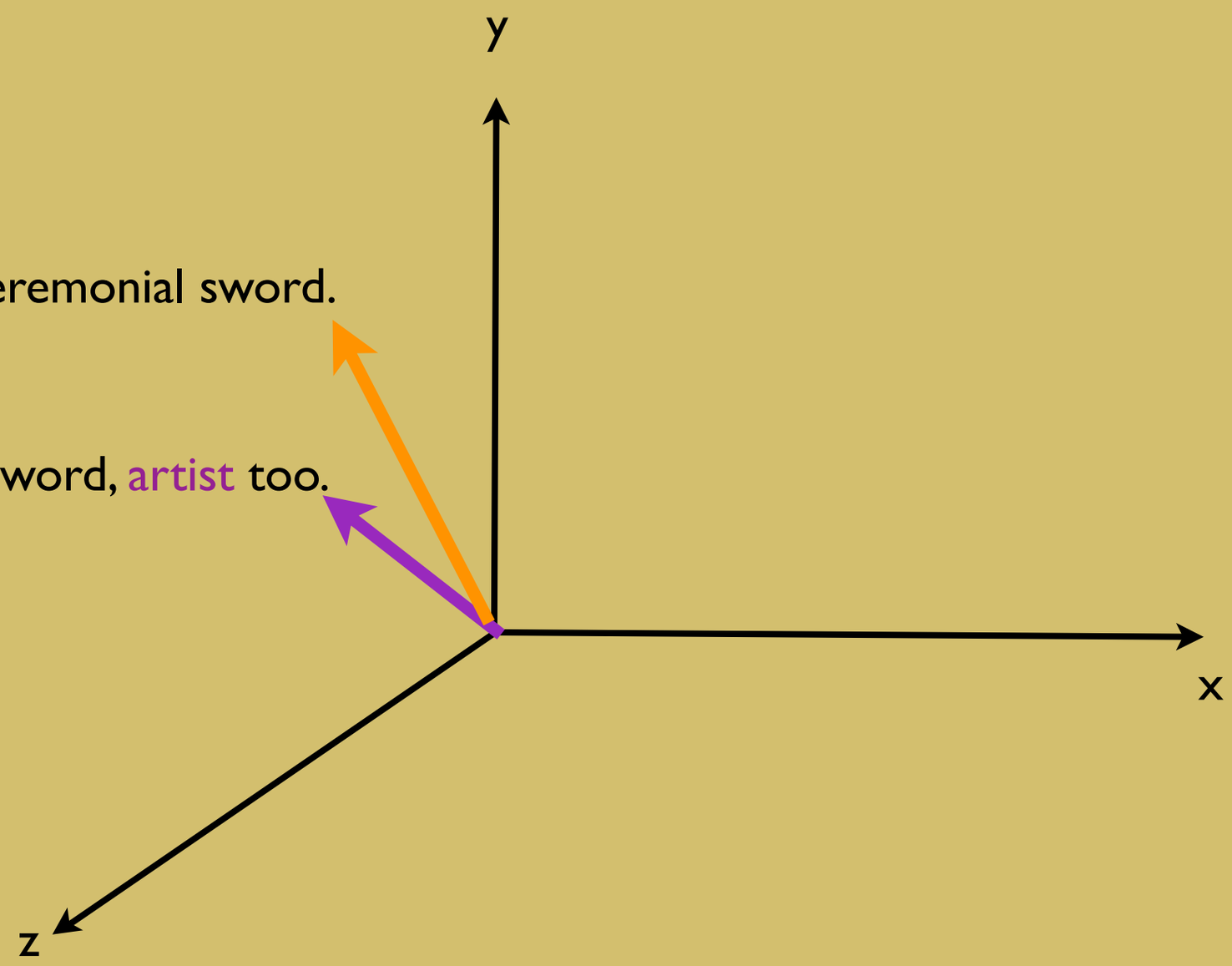
Improves Disambiguation



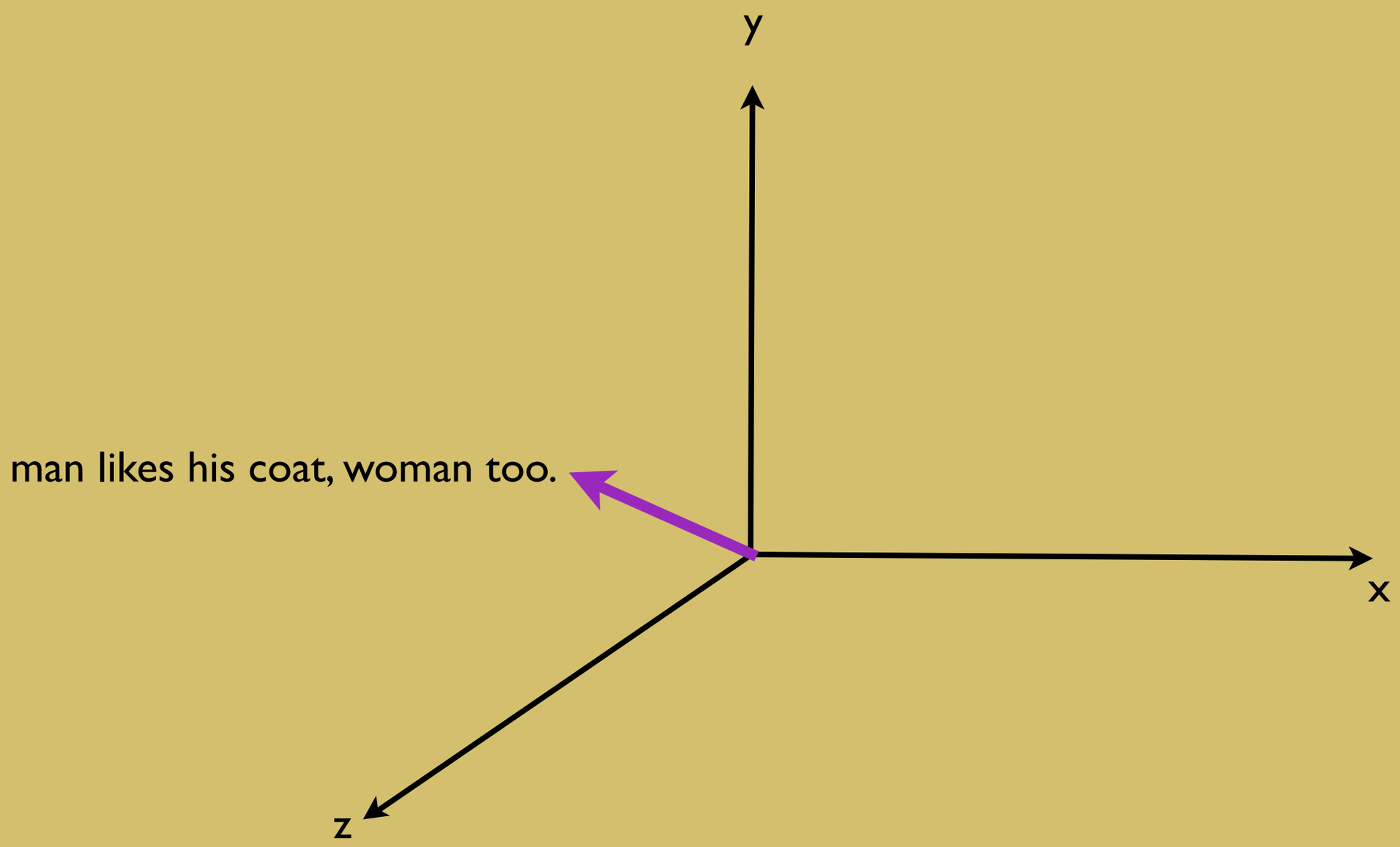
Improves Disambiguation

man sketched ceremonial sword.

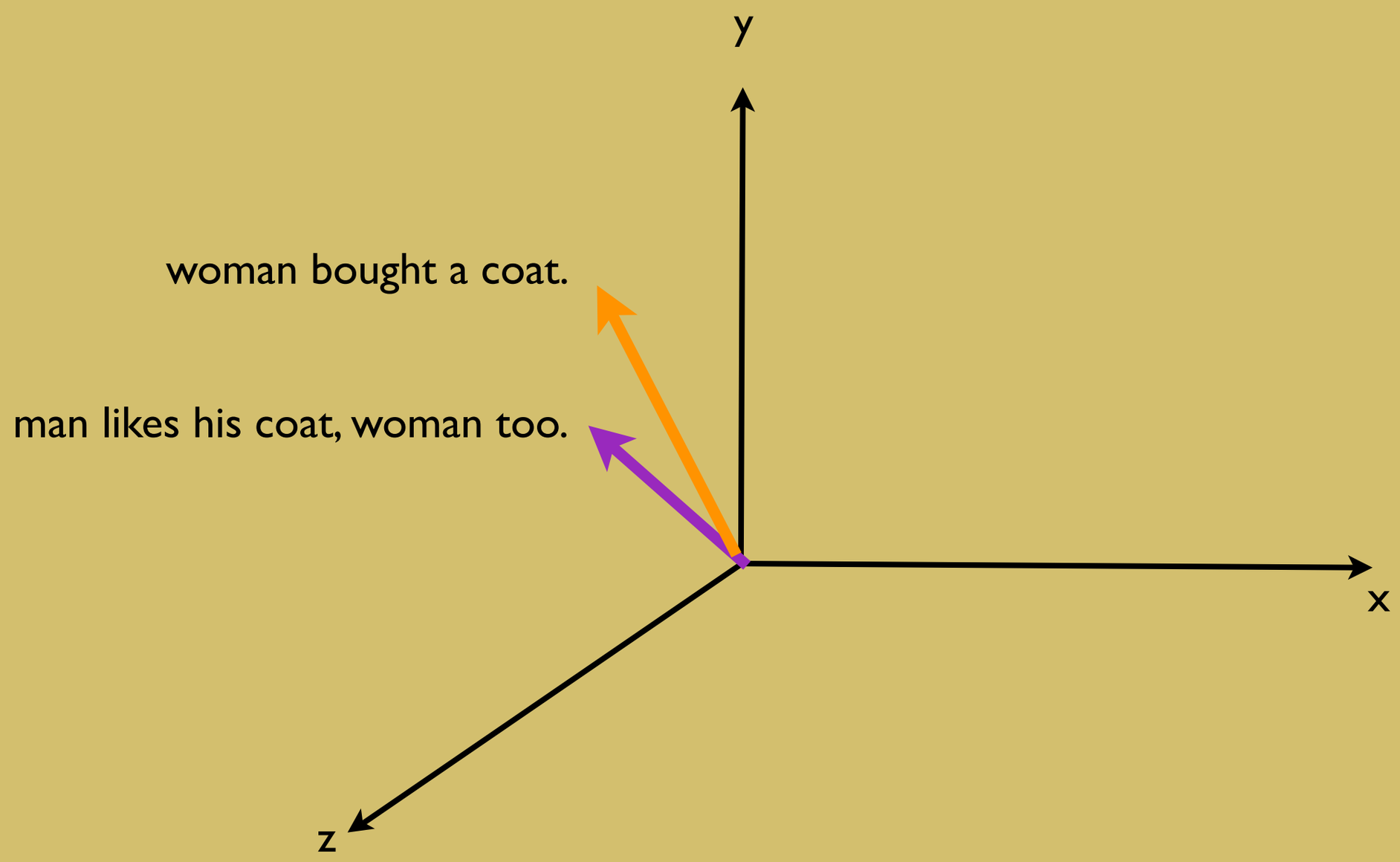
man drew ceremonial sword, artist too.



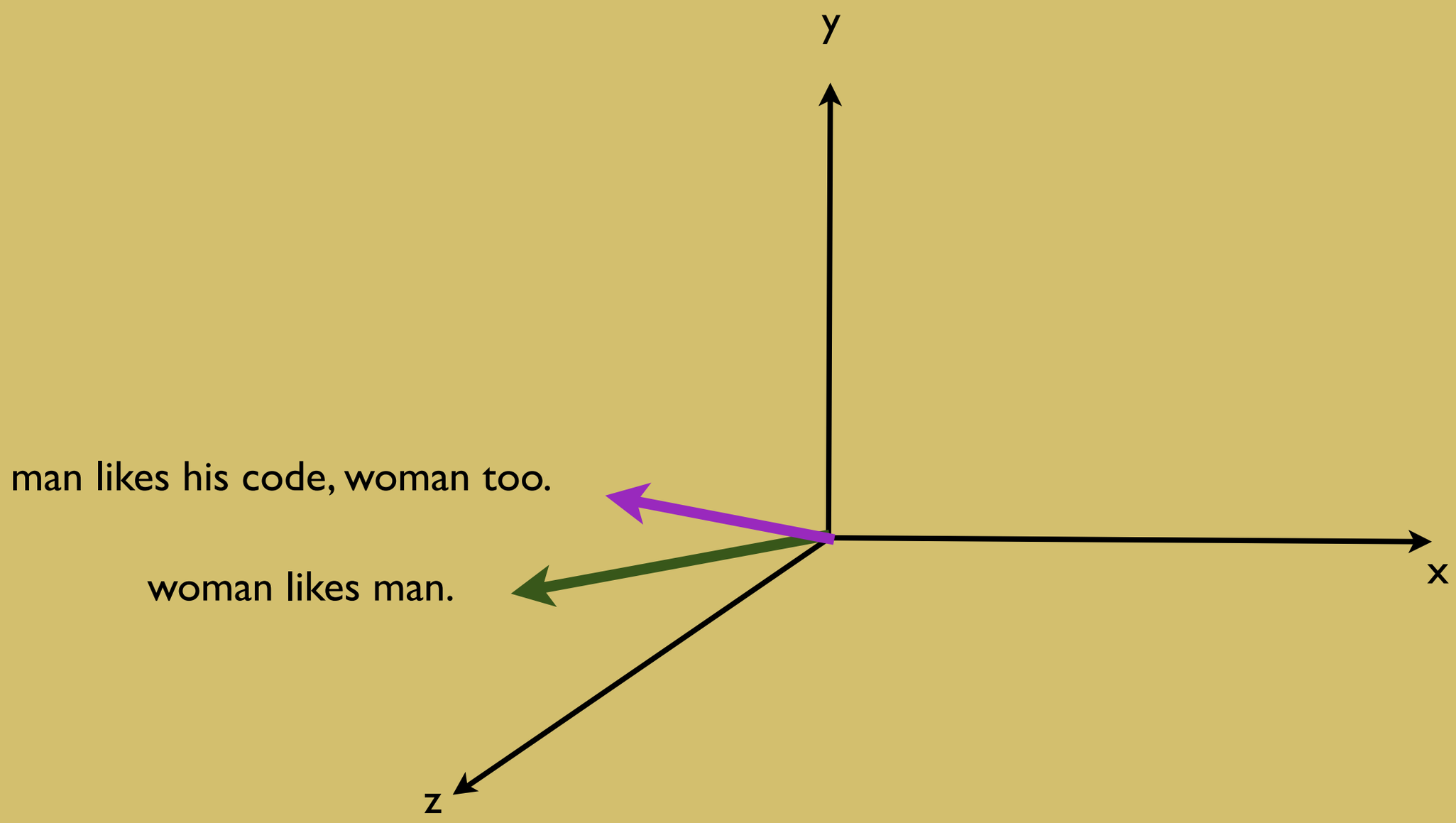
Improves Disambiguating



Improves Disambiguating



Improves Disambiguating



Improves Entailment

KL($\overrightarrow{\text{a cat miaowed}}$, $\overrightarrow{\text{an animal made a noise}}$)

KL($\overrightarrow{\text{cats run across roads}}$, $\overrightarrow{\text{animals move.}}$)

Degrees of Inclusion

Improves Entailment

KL(a cat miaowed, a dog did too. → an animal made a noise →)

KL(cats run across roads, dogs too. → animals move. →)

Increase/Decrease
Degrees of Inclusion

Evaluation

	race	stands
	swm1	swm2
man runs	0.88	0.78

UnResolved

man runs, governor does-too.		0.91
man runs, athlete does-too.	0.92	

Resolved

man runs, governor does-too.		0.98
man runs, athlete does-too.	0.95	

Large Scale Evaluation

Wijholds, Sadrzadeh, JoLLI 2019, NAACL 2019

export boom and economy does too	gun boom and cannon does too
export prosper and economy does too	gun thunder and cannon does too
export thunder and economy does too	gun prosper and cannon does too

frequency search

economy: occurs with boom, but much more with *prosper* than *thunder*.
cannon: occurs with boom, but much more with *thunder* than *prosper*.

Disambiguation with Transitive Elliptical Phrases

	MLELLDIS	`word2vec`	Count Based
UnResolved	Verb Only Vector	0.274	0.078
	Verb Only Tensor	0.060	0.108
	Additive	0.292	0.040
	Multiplicative	0.068	0.206
Resolved	**Multiplicative**	0.213	**0.391**
	Additive	**0.298**	0.078

Disambiguation with Transitive Elliptical Phrases

		CB	W2V	GloVe	FT
	Verb Only Vector	.4363	.2406	.4451	.2290
	Verb Only Tensor	.3295	.4376	.3942	.3876
UnResolved	Add. Linear	.4416	.2728	.3046	.1409
	Mult. Linear	3250	-.0123	.1821	.2928
Resolved	Add. Non-Linear	.4448	.3275	.3262	.1399
	Mult. Non-Linear	.5029	.2087	.2446	.0440
Resolved w Tensors	Best Tensor	**.5385**	.4621	.3688	.4937
	2nd Best Tensor	.5263	.4544	.3581	.4652

A Problem:

What is the algebra of discourse and how does its interface to tensor semantics work?

(I) Copying and Moving

- Jaeger, A multimodal analysis of anaphora with ellipsis, 1998.
- Morrill & Saladrigas, Generalising discontinuity, 1996.
- Hendriks, Ellipsis and multimodal categorial type logic, 1995.
- M. Kanovich, Kuznetsov, Schedrov, Lambek Calculus with a Relevant Modality, 2017.

(II) Bidirectional Implication

- Kubota and Levine, Gapping and like-category coordination, 2012.
- Kubota and Levine, Pseudo gapping as pseudo vp-ellipsis, 2017.
- Jaeger, Anaphora and Type-Logical Grammars, 2006.

Modal Residuated Monoids

$$(M, \cdot, 1, \leq, \backslash, /, \Diamond)$$

Copying

$$a \leq \Diamond a \cdot a \qquad a \leq a \cdot \Diamond a$$

Moving

$$\Diamond b \cdot a \leq a \cdot \Diamond b$$

$$a \cdot \Diamond b \leq \Diamond b \cdot a$$

Bidirectional Implication

$$\frac{[N : A]_i \quad \ldots \quad M : A|B}{M\ N : B}\ E|, i$$

$$(M, \cdot, 1, \leq, \backslash, /, |)$$

New Rule

$$a \cdot b \cdot a|c \leq a \cdot b \cdot c$$

Modal Residuated Monoids are Bidrectional!

Definition

$$a|c := \Diamond a \setminus c$$

Lemma

$$a \cdot b \cdot \Diamond a \setminus c \leq$$

$$a \cdot \Diamond a \cdot b \cdot \Diamond a \setminus c \leq$$

$$a \cdot b \cdot \Diamond a \cdot \Diamond a \setminus c \leq$$

$$a \cdot b \cdot c$$

Monoid Grammer

$\mathcal{F}$

Tensor Algebras

Strongly Monoidal Functor

Modal Monoid Grammer

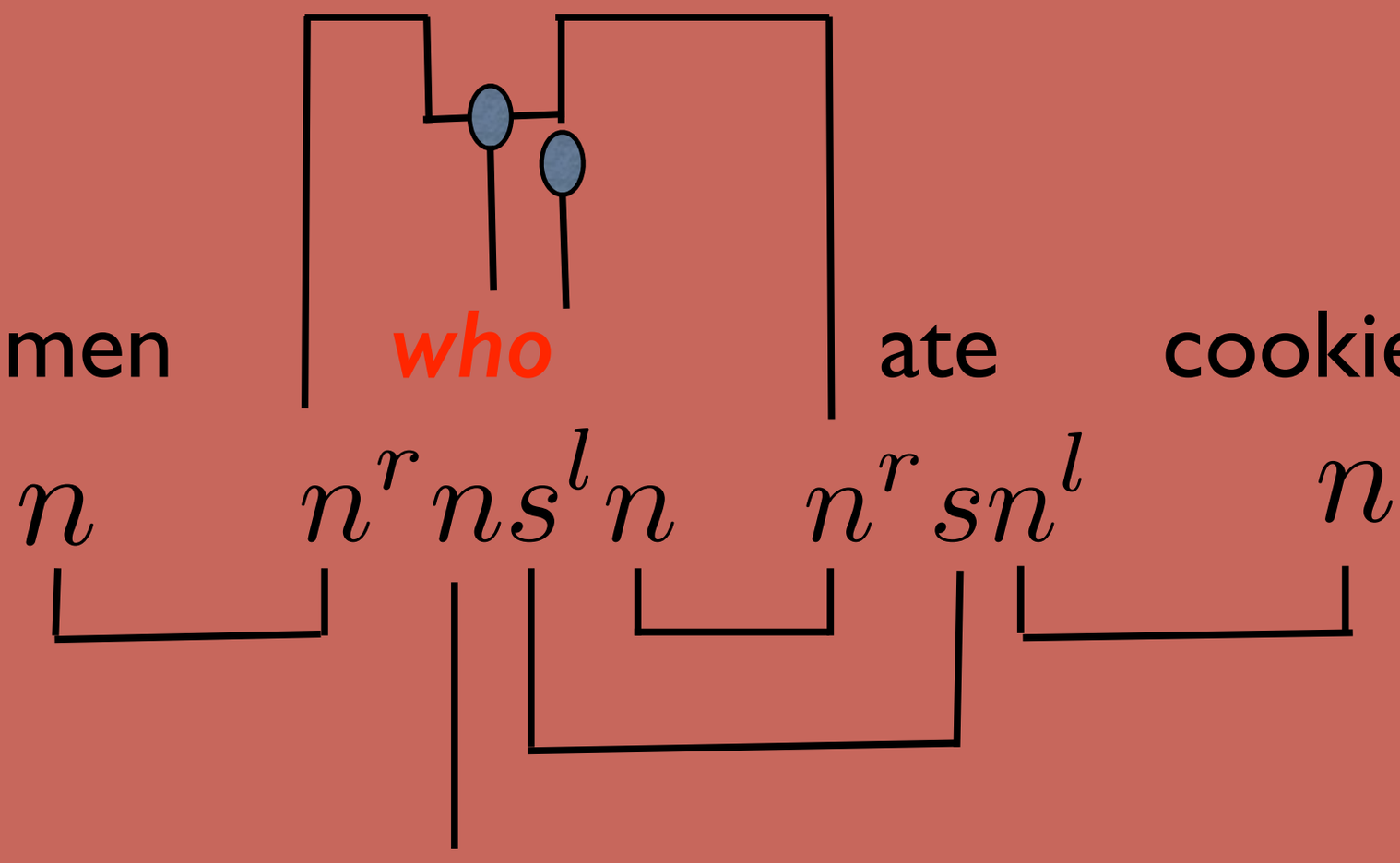
$\mathcal{F}$

?

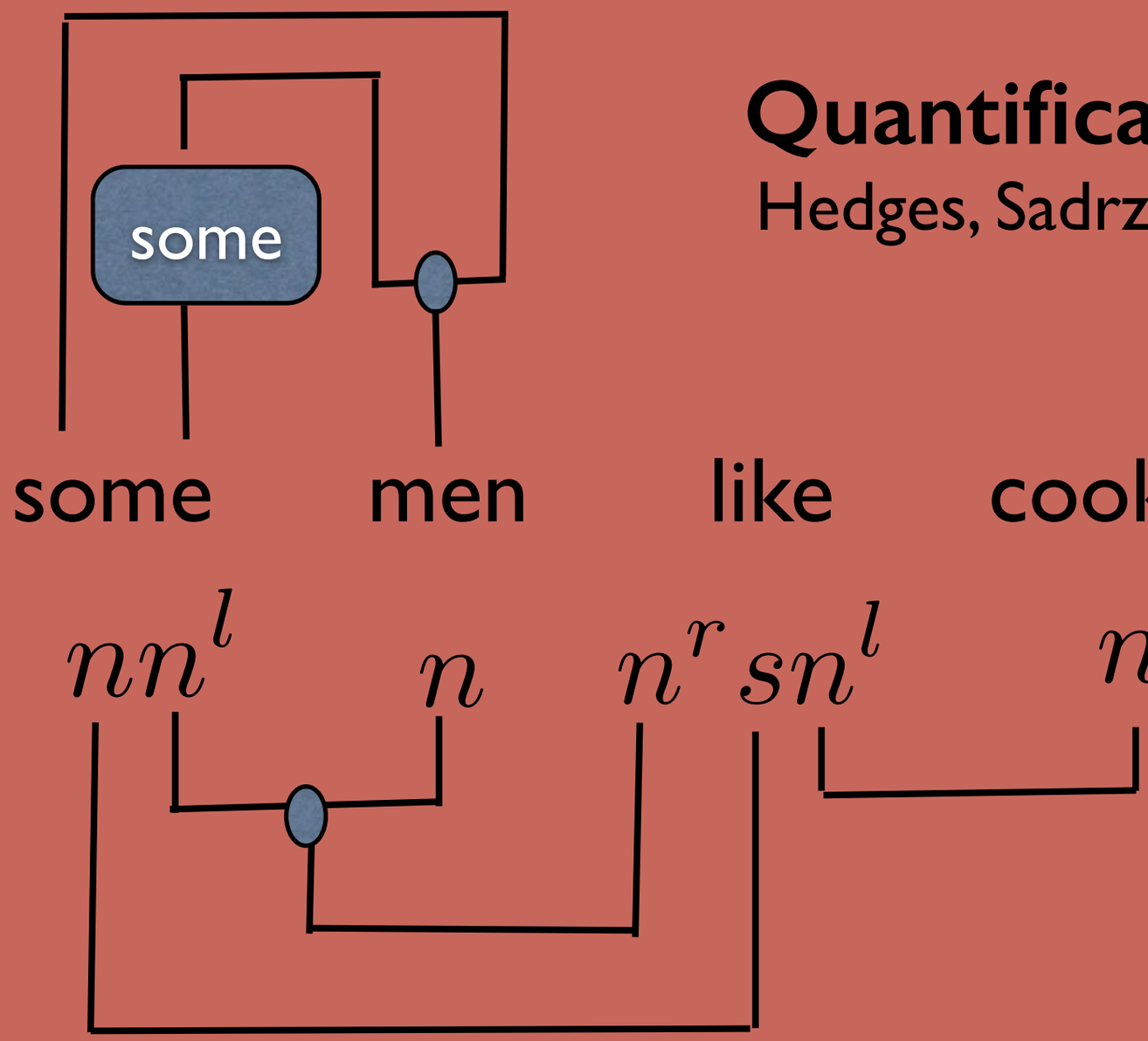
Tensor Algebras

Relative Pronouns via Frobenius Algebras

Clark-Coecke, Sadrzadeh MoL 2013, JLC 2013, JLC 2014



$$\overrightarrow{\text{men}}^T \odot (\overrightarrow{\text{ate}} \times \overrightarrow{\text{cookies}})$$



Quantification via Bi Algebras

Hedges, Sadrzadeh QPL 2016, MSCS 2019

some men like cookies.

$nn^l \quad n \quad n^r s n^l \quad n$

$$(\overrightarrow{\text{some}} \times \overrightarrow{\text{men}})^T \odot (\overrightarrow{\text{like}} \times \overrightarrow{\text{cookies}})$$

Failure of Frobenius for Copying

Wijnholds, Sadrzadeh, CAPNS EPTCS 2018,

Kim likes his code, so does Sandy.

Strict Reading —> Kim likes his code, Sandy likes Kim's code.

Sloppy Reading —> Kim likes his code, Sandy likes Sandy's code. x

Sloppy = Strict

Modal Monoid Grammer

Kanovich, Kuznetsov, Schedrov
Linear Logic !

$\mathcal{F}$

?

Tensor Algebras

$$!A \mapsto A \oplus A$$

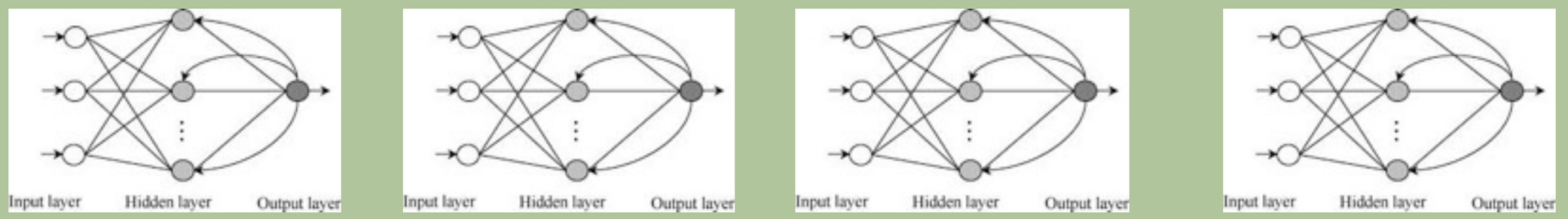
$$\oplus \equiv \otimes$$

A Promise:

Learning the tensors using neural nets

Learning the Tensors: Neural Networks

Dogs Chase White Cats



Wijnholds, Sadrzadeh, Clark,
under review

$$T_{ij}^{red} T_{j}^{car} \sim T^{men}, T^{love}, T^{large}, T^{Teenagers}$$

$$T_{ij}^{red} T_{j}^{car} \not\sim T^{sky}, T^{spider}, T^{sofa}, T^{dance}$$

Thank ⊗ you ⊗ for ⊗ listening!